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PREPHYSICS*

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> *"To be sure, it has been pointed out that the introduction of a space-time continuum may be considered as contrary to nature in view of the molecular structure of everything which happens on a small scale. It is maintained that perhaps the success of the Heisenberg method points to a purely algebraic method of description of nature, that is the elimination of continuum functions from physics. Then, however, we must also give up, by principle, the space-time continuum. It is not unimaginable that human ingenuity will some day find methods which will make it possible to proceed along such a path."* ... *Albert Einstein (1936)*

I

Ever since the ancient Greeks, speculations concerning man's place in the Universe have been an ongoing practice within science and philosophy. Whereas the antique and medieval scientists and philosophers, according to the common tradition in history and philosophy of science, interpreted Nature in subjective terms, it was the ingenious insight of Galileo to emphasize the *method* of physics (mathematization and experiment), which was to secure the objectivity of the practice of physics. This seemed to detach the measuring Subject from the unique and egocentric position it once enjoyed in scientific and philosophical thinking. It was Galileo who fathered the modern concept of mathematized natural science. He tried to achieve exactness and rational objectivity through the use of *mathematics.* According to Galileo, as he writes in the *Saggiatore* (1623), "(philosophy) is written in that great book which ever lies before our eyes, I mean the universe, but we cannot understand it if we do not first learn the language and grasp the symbols in which it is written. This book is written in the mathematical language" [1].

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This bold statement has proved useful ever since it was written. Physicists started to formulate mathematically informally observed regular phenomena, like a falling stone or the flight of a cannonball. This method involved, inevitably, a high degree of scientific idealization. Physicists emphasized the construction and detailed study of scientific models. It was thought that all phenomena must be describable in terms of the mathematical method. The classical highlight was Newton's *Philosophiae Naturalis Principia Mathematica* (1687) [2].

The result was, as far as the practice of theoretical physics is concerned, the adoption of *calculus* (including the metaphysical idea of infinitesimals) as the mathematical tool of the theoretical physicist.

The method was initially formulated by Newton and Leibnitz. Later it was perfected by Bolzano and Cauchy. Nature came to be regarded as being written in the language of mathematics and the whole Universe became understood as a mechanism, as a kind of a "universal clockwork," the blueprint of which is written in the language of calculus. In Kuhn's terminology: the classical calculus became, as far as mathematics is concerned, the

paradigm

of classical physics, which, in general terms, was characterized by the commitment "to the same rules and standards for scientific practice" [3]. The paradigm of Galilean physics is characterized by the adoption of mathematics and the experimental method.

The Galilean method was assumed to have secured the *objectivity* of the practice of physics, since "that commitment and the apparent consensus it produces are prerequisites for normal science, i.e., for the genesis and continuation of a particular research tradition" [4]. For a long time this seemed to be the correct attitude to take; the Subject could safely perform the practice of physics by virtue of this paradigm in order to exhibit facts of the material Universe. This was the highlight of the era of the "classical" paradigm in physics regulated by the Galilean method, culminating with the formulation of Einstein's Special Theory and General Theory of Relativity.

However, in the beginning of this century, Planck formulated the idea of *quanta* in physics (1900). Within three decades, the problem of the Subject erupted once again. It was assumed that quantum mechanics forced the detached Subject back into physics; also, doubt concerning the space-time continuum, presupposed in the paradigm of classical physics, became—as a result of the introduction of the quantum theory—more manifest. Quanta introduced discreteness into physics and, as a result, thoughts concerning the necessity of a change of paradigm in physics, i.e., an essential discretization of physics, surfaced every now and then. A number of approaches to the problem of discrete physics were exhibited, e.g., in Bastin's book *Quantum Theory and Beyond* [5]. However, nothing substantial erupted which barred the creation of a novel paradigm.

To achieve such a change would have required a change of paradigm in physics. It amounts to a change in the paradigm (structure) of mathematics *used* in theoretical (mathematical) physics within the practice of physics. But, one may ask, what is really to be understood by a paradigm? In order to attempt to answer this question, one can begin by noting that usually, today, physics is divided into *experimental* physics and *theoretical* (and mathematical) physics. However, it has not been appreciated that there is a mathematical structure regulating the practice of theoretical physics. One can say that physicists (and mathematicians), usually, only have an implicit understanding of this structure. It is usually not an explicit part of a theoretical physicist's understanding. Nevertheless, a paradigm is always present as a tacit component in the competence exhibited by a theoretical physicist.

Now, when attempting to formulate a discrete physics, one is simultaneously dealing with an attempt to change paradigm in physics; thus, in order to accept a discrete physics, one must additionally accept the implied change of paradigm. This forces the proponent of discrete physics to be able, upon request, in principle, to exhibit the structure of the paradigm. One is to be cautious with this distinction, since the notion of a paradigm in physics amounts to a novel element in the practice of physics, in addition to the previous elements of theory and experiment. This has not been previously explicitly appreciated.

In order to investigate the notion of a paradigm in theoretical physics, one has to begin by grasping the insight that, indeed, there are objects (e.g., events), but that it is *acts* which are real, actual or concrete. Moreover, the acts are immediate acts and the objects are obtained as the *result* of acts. In the terminology of Austin: the immediate acts amounts to *performatives.* When an expression exhibits a performative, it does not *describe* my doing of what I should be said in so uttering to be doing, or state that I am doing it; it is to *do* it. The performative indicates that the issuing of an utterance (expression) is the performing of an action. In uttering an expression, one is *doing* something. We do something in saying and writing something when engaged in tasks of mathematics and theoretical physics: we *judge.* The activity of theoretical (mathematical) physics amounts to performing certain tasks, and to perform a task is to be understood in the sense of it being *immediately* performed. We are not to understand the words "perform" and "practice" in a representational way, as is usually done; to do so leads us astray as far as the point here is concerned. This is a crucial insight.

Thus, there seem to be three major (philosophical) problems connected with the current practice of physics:

1) Where is the place of the Subject in the practice of physics?

2) Ought the theoretical practice of physics to be grounded on a discrete paradigm of physics, instead of the prevailing contemporary space-time arena?

3) How is the notion of "physical reality" to be understood in relation to the practice of physics?

A philosophy of physics, if it aspires to be philosophically complete, as far as the meaning of the practice is concerned, must deal with all three problems. These three problems are not necessarily connected. One can attempt to deal with one, without attempting to deal with the others. However, for the practice of physics to be completely understood relative to the practice, itself, all three questions must be answered—taken together, they belong to a philosophical investigation of the practice understood as a whole; to understand physics as a practice presupposes a holistic conception of the practice. This point is not, yet, explicitly, required to be understood when attempting to formulate a discrete paradigm in physics. Here, the part dealing with experiments as immediate practice and their connection to transcendental reality is not dealt with at all. This part is presupposed to be in order. A physicist working within the paradigm of discrete physics can be equally in the dark concerning explicit understanding of transcendental reality as a physicist working within the continuum paradigm.

The aim, in this paper, is to make one aware that whereas the second point exhibits a genuine methodological task to be performed, the first and the third points only exhibit problems when being the target of a philosophical illusion, brought about by thinking along a representational way of grasping the problem. The first and the third points do not state genuine structural problems. Thus, by a paradigm in physics is to be understood a mathematical framework and the rules connecting this framework to the existing practice of physics [6].

In 1905, Poincaré wrote that mathematics is the mathematical physicist's "special language," indeed the "only language he can speak" [7]. He went on to say that the mathematical physicist uses mathematics not only for calculation but "above all, to reveal to him the hidden harmony of things" [8]. The mathematics referred to here, is, in contemporary practice of theoretical physics, usually taken to be "classical" mathematics (continuous mathematical constructions). With the arrival of the computer, a novel dimension in the history of physics has erupted. The time has come when we can seriously start thinking of replacing part of the work which physicists perform with ordinary mathematical equations (within the established paradigm) with discrete physics, which enables one to construct (computer) programs having (i.e., exhibiting) the behavior intended for the expert programs developed in the area of research called Artificial Intelligence. Thus part of what is currently called "theoretical physics" can, as a result of adopting the paradigm of discrete physics, be understood as belonging to what—in Dijkstra's terminology—is called "computing science" [9].⋆ The switch of terminology from "computer science" to "computing science," reflects the insight of the presence of the transcendental Subject when programming is performed. As Polanyi has pointed out, the internal workings of the computer can, of course, be completely understood in terms of physical laws. What cannot be so explained is the computer's program. To explain the program requires reference to the *purpose* of the program [10].

The paradigm of discrete physics is also characterized by its adoption of *mathematics* (McGoveran's ordering operator calculus), while computing science essentially makes use of computer *programs.* These have usually been regarded as conceptually distinct practices; thus *classical* set theory and programming languages are conceptually distinct. Due to the work of Martin–Löf in grounding mathematics, we now know that this is not necessarily the case. This requires one to understand Martin–Löf's far-ranging insight that immediate mathematical practice exhibits a systematic distinction between *judgments* and *propositions.* This insight can be transferred to the practice of theoretical physics. However, it requires one to understand the connection between mathematics (constructive set theory) and computer programs, in order to understand the *purpose* of the mathematics used in discrete physics. In discrete physics, consequently, there is to be a connection between the notion of "mathematics" and the notion of "program," in agreement with the insight of Bishop [11].

The two methodologies of mathematics and programming have been regarded as conceptually distinct, as is easily seen in relevant literature. Furthermore, in current literature a sharp distinction is made between "programming"—which has not, for the most part, been understood as a mathematical practice—and "computer science"—which certainly has, as exhibited, e.g., in the work of mathematicians on graph theory, automata, combinatorics and formal languages. In fact, however, they are not conceptually distinct! Grasping this point requires a novel understanding of the method of mathematics—the method of Martin–Löf [12].

The method of Martin–Löf is not concerned with a theory of practical everyday use, but with a theory (paradigm) for understanding the practice of constructive mathematics. Alternatively, one can say that Martin–Löf's Intuitionistic Theory of Types (Sets) exhibits the *logical form* of the paradigm of mathematical language. As Martin–Löf said in a letter to Beeson: "I have been searching for a system which makes good sense, not only as an object of metamathematical study, but in its own right; one that stands on its own feet, so to speak" [13]. Achieving this, became, for Martin–Löf, the task of restoring the computational meaning of the well-known mathematical notions such as *function* and *proof.* As Martin–Löf points out, it was Brouwer, who realized the necessity of so doing: the true source of the uncomputable functions of classical mathematics is not the axiom of choice (which *is* valid intuitionistically) but the law of excluded middle and the law of indirect proof [14].

⋆ Of course, it is essential that the "computer" be realizable.

The intention of Martin–Löf's investigations is to make the Subject aware of the *common* structure of mathematics and programming languages. The genuine source of the difference between constructive mathematics and programming does *not* concern the primitive notions of either, since they are the *same,* but lies in the unreflective use of (1) program forms required in order to be read and executed by the computer, and (2) on the part of constructive mathematics in the fact that computational procedures (programs) are usually left *implicit* in the proofs (computations). Consequently, considerably further work is needed in order to exhibit them in a form fit for mechanical execution. Thus, Martin–Löf writes in his *Constructive Mathematics and Computer Programming* that "the whole conceptual apparatus of programming mirrors that of modern mathematics (set theory, that is, not geometry) and yet is supposed to be different from it. How come? The reason for this curious situation is, I think, that the mathematical notions have gradually received an interpretation, the interpretation which we refer to as classical, which makes them unusable for programming.... Now, it is the contention of the intuitionists (or constructivists, I shall use these terms synonymously) that the basic mathematical notions, above all the notion of function, ought to be interpreted in such a way that the cleavage between mathematics, classical mathematics, that is, and programming that we are witnessing at present disappears" [15].

These insights of Martin–Löf are reflected in the mathematics used in discrete physics. Thus one of the primary tasks of discrete physics (there are, indeed, many others) is to exhibit the common logical structure of theoretical physics, computer languages and constructive mathematics.

For example, the notion of a *function* is to be understood in the sense of a *method* to be applied in order to achieve a result within the paradigm of discrete physics. A function is *not* to be understood as a relation between arguments and value. A function is defined by providing *rules* (the method) for its calculation. As far as mathematical and computational practice is concerned, these rules amount to the 48 inferential rules of the Intuitionisic Theory of Types. Note that functions are not objects in the metamathematical sense of which it could be proved that they have the property of yielding unique values; rather, that functions yield unique values is to be *understood.*

The notion of *verification* (proof, computation) is to be understood in the same way as Martin–Löf. This amounts to understanding verification as a performative. One performs an immediate verification when one computes a *result* in theoretical physics. The aim when engaging in formulating the paradigm of discrete physics, in analogy with Martin–Löf's program, is not to formulate a language of theoretical physics for practical (explicit) everyday use, but to formulate a paradigm in physics in order to understand the practice of computation in physics in a more meaningful way.

In the paradigm of discrete physics, a "theory of physics" can be read as a *person program.* Thus, a theory is a piece of information (implicitly) giving instructions concerning what to do in order to attempt to falsify the theory (person program) in question. Consequently, a theory of physics and a person program amounts to the same. It is only a question of preference if one wants to adhere to a more object-oriented mode of language and talk of "theories," or, if one prefers a more subject-oriented mode of language to talk of "person programs." They amount to the same as far as content is concerned, i.e., they are synonymous ways of expressing the same point. Within the paradigm of discrete physics, a formulated theory *is* a person program (implicitly) giving instructions of its own validity when attempting to falsify a proposition by virtue of an experiment (performative).

Problem 1.

In order to show that the results of measurements, indeed, exhibit objective facts, the Subject has to engage in a philosophical (phenomenological) investigation of the practice of physics. It amounts to exhibiting the *practical* understanding *as it is given* in the immediate practice of physics. As far as theoretical physics is concerned, it amounts to understanding the practice of the theoretical devices used in theories of physics. Without a shared commitment to a set of symbolic generalizations, logic and mathematics could not routinely be applied in the community's work. In the terminology of McGoveran, to investigate this practical understanding amounts to investigating the "E–frame" (Epistemological framework) [16]. This amounts, in Husserl's terminology, to grounding (*begründen*) the practice of (theoretical) physics and is, essentially, a descriptive activity. This grounded practice is also the starting point when attempting to formulate a discrete paradigm in physics.

Contemporary practice of physics is usually regarded as consisting of, essentially, two subpractices (computer physics is usually not included in current practice): (1) experimental physics and (2) theoretical physics, where, according to Popper, "[theories] are nets cast to catch what we call 'the world': to rationalize, to explain, and to master it" [17]. What has not been generally understood, however, is that theories are always cast within some paradigm. Contemporary theories of physics are usually cast within the paradigm of continuum mathematics, where the paradigm "is what the members of a scientific community share *and,* conversely, a scientific community consists of men who share a paradigm" [18]. The current mathematical paradigm in physics is assumed to be based on the acceptance of the space-time continuum in the mathematics used (exhibited, e.g., as "renormalization" in quantum field theory or Wheeler's and Hawking's idea of "space-time foam" in quantum gravity).

The task of discrete physics is to change our understanding of the current paradigm. However, one cannot even *attempt* this if one is not already familiar with current practice of contemporary theoretical physics. One must have acquired the competence to engage in the practice of contemporary theoretical physics. In the terminology of Polanyi: what is required is "tacit knowledge" which is learned by doing science and not by acquiring rules for doing it [19]. These practices, as shared examples, must function as *data* for any attempt to engage in a paradigm shift in theoretical (mathematical) physics.

In order to engage in a paradigm shift, the Subject is to formulate the novel paradigm of theoretical physics. This amounts to engaging in what is to be called

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In McGoveran's terminology, it amounts to formulating the R–frame (Representational framework) and the P–frame (Procedural framework). To formulate these two frameworks amounts to formulating a paradigm in physics.

The R–frame "is an abstract formalism consisting of a set of symbols and a set of rules of manipulation" [20]. The logical form of the "rules of manipulation" amounts, when codified, to the 48 rules of Martin–Löf's Intuitionistic Theory of Types. To engage in formulating the R–frame is to engage in *syntax* in the terminology of prephysics. The activity of syntax is a speculative activity in the sense "that we really do not know what we are talking about," when engaging in this creative task. There are no rules regulating the activity of syntax. It is a speculative (and normative) activity. It is the absence of regulative rules which makes it possible to call syntax a creative activity.

What is still missing are explanations relating the observations (performed within the E–frame) and the symbols of the R–frame, which then, through recursion, serves to establish the relation between the E–frame and R–frame, until a sufficient level of agreement concerning accuracy is achieved or the paradigm fails (a la Kuhn) [21]. This explanation establishes the procedural framework, or the P–frame. In prephysics, the activity of formulating the P–frame is called *semantics.* It is a higher-order activity which, in a logical sense, can only be performed after the formulation of the R–frame is completed. In semantics we explain the E–frame by explaining the relation between it and the R–frame formulated in syntax. The explanation itself amounts to the rules regulating the connection between the old E–frame and the R–frame, thus supplying novel meaning to the E–frame.

The P–frame can also be understood as exhibiting, within the novel paradigm, a *translation manual* between the expressions occurring in the old E–frame, thus determining how far they can be given meaning within the novel paradigm. Alternatively, it can be understood as a *modeling* of the E–frame. In setting up the paradigm, we give, at the same time, a manual for translating between it and the ordinary forms of expressions used in practice (E–frame), and a model for these ordinary forms of expressions. The aim with semantics is to achieve a reflective equilibrium of understanding the practice of physics when performed according to the paradigm of discrete physics. Note that by semantics is not meant any representational, as is the case in the model-theoretical sense of semantics. The sense in which the word "semantics" is used in prephysics is not meant to be a branch of mathematics (like logical semantics, or its technical twin, model-theory); it is the activity of describing the relation between the E–frame and the R–frame.

In the research program of attempting to formulate a discrete physics, one is also concerned with establishing a novel paradigm in physics; or, as one could also formulate the point, we are concerned with establishing a novel paradigm of theoretical (mathematical) physics. Having successfully formulated the R–frame in syntax and described the connection between the E–frame and the R–frame by formulating the P–frame in semantics, one is to have achieved *theoretical understanding* of the practice of informal theoretical physics. Note that by the expression "theoretical" in the context of theoretical understanding is not meant "theoretical" as the expression is used in connection with, say, theoretical physics. In the sense the notion of "theoretical" is used here it purports, or attempts, to be a paradigm in the practice of theoretical physics. It cannot be conceptually separated from this practice because the paradigm determines the practice of theoretical physics to be what it is; to be able to formulate the point of the paradigm is to exhibit theoretical understanding of the practice of theoretical physics.

By engaging in prephysics one cannot exhibit the *sense* of the practice of physics, when the notion of sense (meaning) is understood as standing for what Frege called *Sinn*; that is, one cannot make the conceptual distinction between sense and reference in the modeling methodology of prephysics (as is assumed when semantics is understood in the model-theoretical sense). Prephysics establishes rules of meaning (definitions) in the sense of semantical descriptions (the P–frame), but presupposes rules of sense in order to be possible in the first instance at all. The rules of sense amount to the competence to use a natural language in order to grasp the point of the E–frame as a "universal medium of communication" (in Hintikka's terminology) in the first place.

Recall that we are dealing with the *practice* of physics requiring the presence of a transcendental Subject. All thinking presupposes the presence of a natural language as far as grasping the point with regard to the practice of theoretical physics is concerned. Indeed, this is what makes it possible to grasp the universality of physics in the first place. In this sense the transcendental Subject can be equated with natural language. This leads to the insight that *any* practice of physics (performed within any paradigm) exhibits a form of the Anthropic Principle to be called the

Transcendental Anthropic Principle

which, essentially, states that *natural language, being the Universal Medium of Communication, is necessary in order to bring the Scientific Universe into being-as-fact in the practice of physics.* This principle will be treated in more detail below.

Problem 2.

Since the classical period in physics, the idea has become familiar that a physical object is something real, existing outside of the thinking Subject, independent of whether or not the object has been subjected to observation. This has, in fact, on many occasions, been taken to be the criterion for scientific objectivity, since, it is claimed, one cannot attribute to a system at every instant its measurable properties. As a result of the introduction of quantum mechanics in the first half of this century, it seems at first sight that the concept of scientific objectivity has been strongly shaken. For example, one cannot even claim that a wave function has a well-defined meaning unless one explicitly refers to a definite measurement. Furthermore, it looks as if the result of a measurement is intimately connected to the *acts* (Bridgman: operations) of the Subject performing it, and thus, as if quantum mechanics drives one towards a complete subjectivism in the practice of physics.

One can formulate the problem like this: quantum mechanics is fundamentally about "observations." This is usually understood as implying a separation of the Universe into two parts, a part which is observed (*res extensa*) and a part which does the observing (*res cogitans*), if we use the terminology of Descartes. However, since Galilei and Newton it has been a basic requirement that physics ought to be objective. How is one to cope with this enigma? What is one to understand by the term "objective"? How is one to provide meaning to this term? The usual way out of the dilemma is to adhere to a *realistic* interpretation of theoretical physics by presupposing some kind of space-time continuum.

This cannot be done in discrete physics, which implies that one has to cope with the dilemma in some other way. Here we meet the first difficult insight:

> *To understand that physics primarily amounts to an immediate practice, and is only secondarily concerned with laws of nature and physical objects.*

To grasp that physics is essentially an immediate practice is more fundamental than to understand physics as concerned with certain laws and objects. Provided a measurement (experiment) of physics is made up of three discernible components: Object + Apparatus + Subject, as a combined and unique whole, then the philosophical problem becomes the task of grasping how a measurement provides objective knowledge of fact. By "objective" is to be understood the validity of a *result* of a measurement for any Subject participating in (performing) the practice. This amounts, essentially, to understanding a measurement in the sense of a *performative*.

To grasp the point that the immediate practice of measuring is a closed whole exhibiting *objective* knowledge of fact, can be regarded as the main puzzle in connection with measurements in "classical" physics, as well as in "classical" quantum mechanics. Both practices are "classical" relative to the mathematical paradigm applied. To correctly understand the objective semantical *force,* in Frege's terminology, of the immediate practice of measuring, amounts to grasping the objective self-evidence, or the *meaning,* of measurements in physics. And, as Wheeler emphasizes, "[no] feature in all physics voices more insistently the message 'meaning is central,' than the elementary quantum phenomenon" [22].

The philosophical task we are confronted with thus becomes to grasp the meaning of the *immediate* practice of performing computations and measurements in (discrete) physics. This must be somewhat qualified. It amounts to understanding these practices as exhibiting performatives. It is what the Subject actually *does,* i.e., the immediate acts (Bridgman: operations) of computing and measuring, that is real, actual or concrete. To be more precise, by an act is to be understood an act of *judging.* We shall return to this important insight below; it is enough for the moment to emphasize that the Subject is to grasp the point with regard to judgments; to be able to use public judgments in different informal practices of physics. That is, the Subject is to *understand* a certain practice in order to participate in that practice, to be able to exercise the faculty of judgment.

The Subject is to break into the circle of understanding by, in one way or another, achieving *practical competence* (the E–frame) to perform certain tasks, like computing and measuring. To exhibit practical competence to perform a certain task amounts to having practical understanding of the task; to be able to perform it. Here, it is simply a question of somehow achieving the practical skill to perform certain tasks. It is not, primarily, a question of describing verbally what is done. Practical understanding by the Subject is exhibited in having the competence to achieve results when engaging in the practice of performing computations in theoretical physics.

This competence is presupposed in order to be able to grasp the very point of engaging in prephysics. In other words, the primary task of practical competence (the E–frame) is to make the Subject grasp to what inductive reasoning amounts. Thus, prephysics makes us aware that *induction turns out to be the same concept as recursion.* They both amount to an immediate practice exhibiting practical competence. As an example of the presupposition of practical understanding (the E–frame), one can give the way a judgment of the form "a $\in$ N" is introduced by Martin–Löf: "a has value either 0 or $a_1\prime$, where $a_1\prime$ has value either 0 or $a_2\prime$, etc., until eventually, we reach an element $a_n\prime$ which has value 0" [23]. The point here is to emphasize that the task to be performed terminates after a finite integral number of steps and that this statement is not a metamathematical statement.

Here one clearly recognizes the necessity to have the competence of induction (recursion). It is crucial. As Poincaré said,

> "...induction, that is, demonstration by recurrence... imposes itself necessarily because it is only the affirmation of a property of the mind itself" [24].

This was also Brouwer's position. In defending this position (of Poincaré and Brouwer), Weyl writes:

> "When Poincaré claimed that *mathematical induction* is for mathematical thought an ultimate basis that cannot be reduced to anything more primal, he had in mind precisely the processes, of composition and decomposition of numerals, that Hilbert himself employs in his contentual considerations and that are completely transparent to our perceptual intuition. For after all Hilbert, too, is not merely concerned with, say, 0' or 0''', but with any 0'' ···', with an *arbitrary concretely given* numeral. One may here stress the 'concretely given'; on the one hand, it is just as essential that the contentual arguments in proof theory be carried out *in hypothetical generality,* on *any* proof, on *any* numeral. This, of course, is not to be taken as an objection, for the procedure of the 'one after the other' can appeal to unshakable intuitionistic evidence; but, evident and primal though it be, we may nevertheless give it expression—not by formulating it as an 'axiom,' but simply by describing its concrete use—making its self evidence and primal quality explicit, and we are no doubt justified in seeing in it the characteristic mark of contentual *mathematical* thought" [25].

The important point to grasp is that induction essentially amounts to immediate practical recursive competence by the Subject. Practical understanding is exhibited in the competence to know *how* to do something practical (the E–frame), thus logically preceding theoretical understanding (this preceding is not a "preceding" in a temporal or empirical sense). As one cannot conceptually grasp *what* the Subject is performing logically distinct from *that* it is being performed, one cannot conceptually separate knowledge of the Universe (physics) from the method of knowing (implicitly or explicitly) *how* to achieve this knowledge, since the factual knowledge is established as the *result* of having performed a (repeatable) measurement. When immediate practice of physics is taken as fundamental, there is, in the end, no distinction between knowing *that* something is the case, and knowing *how* to reach this fact.

Problem 3.

The methodological task the Subject is confronted with in order to formulate and explain the point of discrete physics is to set up a *code* by engaging in *syntax* (the R–frame) and *semantics* (the P–frame). To engage in semantics is to engage in explaining the relation between the E–frame and the R–frame formulated in syntax; that is, semantics amounts to normatively prescribing the use of the expressions of the E–frame within the paradigm. Note that semantics comes last. We retain the idea with semantics coming last, to the extent that there are three discernible components of the paradigm corresponding to its three parts in logical semantics. We can differ between (1) object-valued and type (set)-valued functions (the E–frame), (2) objects and types (sets), i.e., symbols and rules of inference (the R–frame), formulated in syntax, and (3) the semantical part (the P–frame), which can be divided into a formal (stipulatory, mechanical) part and a nonformal (teleological, nonstipulatory) part.

The formal part consists of symbols, like, e.g., the natural numbers, or the symbols for length **l**, time **t** and mass **m**, which are already fully evaluated: if one evaluates the value of a formal symbol, one gets the value back. A symbol which is arbitrarily formed need not, necessarily, have a value relative to the paradigm, but *if* it has a value then that value is necessarily canonical. This is why, for example, such symbols (those in the paradigm) amount to formal expressions. The nonformal part consists of the inferential rules *used* (implicitly) in the E–frame when performing practical tasks in discrete physics. The inferential (nonformal) part can be called the *teleological* part of semantics, because the Subject, in the practice performed within the paradigm of discrete physics, always tends to *use* (implicitly or explicitly) these rules in order to perform the computational task which the Subject set out to achieve. It is important to grasp that the Subject can apply these rules in computational tasks of theoretical physics without being able to *formulate* these rules in an explicit way. The logical form of the inferential rules, when formulated, exhibits itself in the inferential rules of Martin–Löf's Intuitionistic Theory of Types (Sets).⋆ When performing theoretical tasks within the paradigm of discrete physics (also continuum physics), it is important to grasp that one is concerned with a single mechanism from which no one component can be removed without the others losing their nature. This is what makes the practice into a paradigm.

There is a precise rigid order when the Subject is to break into the circle of understanding discrete physics. To engage in syntax and semantics is a genuinely speculative activity. One could compare it with the moment when, after staring at a group of people playing a card game (the E–frame), with growing bewilderment and perplexity, something clicks, and all their operations with the cards fall into place. The Subject grasps what is done in these operations. Formulation of the R–frame and the P–frame does not amount to something that one can passively record from the E–frame. If it were just a question of passive recording, then the Subject would already know the method which is to be exhibited, since the Subject would *already* apply the method in order to record the agreed upon facts occurring in the E–frame.

⋆ Note that this requires reading Martin–Löf's inference rules as pertaining only to finite domains.

Syntax and semantics is to provide a formal language and an explanation of this language which gives a *codification* of the informal concepts and rules used in the practice of theoretical physics. The formal language makes it possible to theoretically *exhibit* the meaning of these concepts and rules as they are used in the practice of physics. The expressions used in informal practice are translated in the language. Since the language is intended as a codification, one should not try to understand the point of the expressions and rules of the language through the translation. It is rather the other way around. The paradigm thus formulated provides the possibility of giving novel meaning (understanding) to the practice of theoretical physics. A practice of theoretical physics is never a conceptually "blind" practice as far as the task is concerned, since it is guided by the paradigm used. This, however, is not the case when engaging in syntax and semantics.

Here one can give an analogy with a machine (the E–frame) that has come down through several centuries. There are a number of people who can run this machine, some of them very skillfully. This would correspond to Kuhn's notion of "puzzle solving" or, alternatively, "research program," in the terminology of Lakatos. Lately, the machine has been put to use in unforeseen circumstances. Now it doesn't work properly; e.g., the Subject is faced with the conceptual separation of relativity theory and quantum theory. The result is that doubts arise whether some of the controls of the machine do anything essential, or whether they are indeed harmful or create havoc in the running of the machine in the new circumstances (e.g., space-time continuum), although they were harmless before; thus, it becomes urgent to understand the machine more profoundly, but *this* task is not just a descriptive undertaking. If the Subject is able to formulate in syntax any principles about the running of the machine, one may want to design new components which exploit these principles more effectively and improve the machine's performance. If we call the syntactical step from seeing just the physical operations (the E–frame) to grasping *what* is being done in the practice of physics *abstraction* (as Martin–Löf does), then we can say that we know of no laws that regulate abstraction; thus, syntax and abstraction amount to the same activity.

Also semantics (the P–frame) is a speculative activity, the aim of which is to establish a reflective equilibrium between the regulative rules (the R–frame) and the practice of measuring in physics (the E–frame). This is performed by describing the point (semantical *force*) of the R–frame formulated in syntax. Such a description cannot be, in the last analysis, performed without the use of natural language. If the Subject is successful in semantics, a novel understanding of the practice of physics, i.e., a novel paradigm, is achieved. One must be aware, however—as Kuhn points out—that "[the] decision to reject one paradigm is always simultaneously the decision to accept another, and the judgment leading to that decision involves the comparison of both paradigms with nature *and* with each other" [26]. Note that one cannot *prove,* by virtue of semantics, that one paradigm is better than another. This can only be *understood.* There is no decision procedure by virtue of which this could be decided. Another way of stating this is to say that when a physicist is to choose between competing paradigms (of continuum and discrete physics), he behaves like a philosopher. In this sense, acceptance of a paradigm always amounts to a *normative* choice: to accept a novel paradigm within physics is to accept a *prescription* concerning practice of physics; thus one realizes that theoretical physics always incorporates a normative component exhibited by the paradigm adopted.

The important point to understand is that it is because the Subject previously has practical understanding and thus—employing the recursive (inductive) competence so achieved—that he can understand the point of prephysics in the first place, and, in addition, decide which of the competing paradigms is more meaningful. Here it is important to point out that semantics (the P–frame) is to bring about an understanding of the paradigm, but that there are certain limits to what verbal explanations can do when it comes to justifying the paradigm. As Martin–Löf has pointed out, "In the end everybody must understand (the point of the paradigm) for himself" [27].

II

Problem 4.

Above we asked "Where is the place of the Subject in the practice of physics?" and "How is the notion of 'physical reality' to be understood in relation to the practice of physics?." We shall now attempt to answer these questions. According to Rorty, "[Discussions ...] in the philosophy of mind usually start off by assuming that everybody has always known how to divide the world into the mental and the physical—that this discussion is common-sensical and intuitive, even if that between two sorts of 'stuff', material and immaterial, is philosophical and baffling" [28]. This position exhibits a category mistake, which, according to Ryle, shows itself as the dogma of the "Ghost in the Machine" [29]. It maintains that there exist both bodies and minds; that there are mechanical causes of corporeal movements and mental causes of corporeal movements. In short, the doctrine assumes that there are physical processes and mental processes conjoined in the same category. This is a mistake. The idea of thinking as a process in the head, taking place in a completely enclosed space, easily provides something "occult." The judging (thinking) Subject is not anything over and above the judgments (thoughts), themselves. As Ryle points out, "(the) belief that there is a polar opposition between Mind and Matter is the belief that they are terms of the same logical type" [30].

This dualistic attitude, as a philosophical standpoint, has, implicitly, been transferred to all interpretations of the role of measurement when read in the light of quantum mechanics. However, it concerns *all* interpretations of measurement in physics; i.e., it concerns also measurements in "classical physics." Indeed, one of the characteristic features of current investigations into the "foundations" of physics is the attempt to provide intelligibility to, say, quantum mechanical structure, by attaching philosophical speculations of the role played by the psychological (empirical) Subject in physics (Wigner). This exhibits a mistake. The mistake concerns the way the notion of a "Subject" is understood, and is reflected in the way the *language* of physics is being understood. Actually, the problem is not a psychological problem, it is a problem of a *semantical* kind. So constructed, the meaning of physics amounts to a semantic thesis; a thesis about what, in general, renders a statement within the practice of physics true when it is true.

The crucial problem shows itself in a certain way of understanding the language of physics, based on the illusion that one, by using language, can provide an *interpretation* of physical reality. This way of thinking can be traced back to two conceptions of logic, which van Heijenoort has named "logic as calculus" and "logic as language" [31]. The conception of "logic as calculus" does not say that logic is like an uninterpreted calculus, but assumes that logic is reinterpretable like a calculus. The conception of "logic as language" amounts to the insight that one cannot get outside our logic, as it were, and its intended interpretation. It amounts to a doctrine of the *universality* (in the sense of *inescapability*) *of logic.* By "logic" one is to understand the point that the Subject, as it were, cannot get outside a practice (when performing tasks in physics) and its intended interpretation, i.e., nothing can be said outside *some* set of formal laws. Another way of formulating this point, is to say that the union of the laws that are possible is inescapable. Hintikka has generalized van Heijenoort's distinction into two basic ways of looking at one's language, which he call's "language as calculus" and "language as the universal medium," where "(as van Heijenoort noted) all logical semantics (model theory) is impossible if the view of language as the universal medium is correct" [32].

The standpoint of "language-as-calculus" leads to the belief that doing semantics (foundations) of physics amounts to providing the *correct* interpretation of, say, quantum theory; thus, philosophy, on this reading, amounts to a metainvestigation. This is, for a number of reasons, a mistaken attitude. By adopting the language-as-calculus way of thinking, the Subject is forced to accept the following theses:

1) Semantical relations are accessible.

2) The Subject can tell what it would be to have different semantical relations.

3) Model theory is possible.

4) Linguistic relativism is not a tenable doctrine.

5) The Subject can reach Reality as such because one can always subtract the influence of language.

6) The construction of metalanguage is possible.

7) Truth as correspondence is possible.

Acceptance of the language-as-calculus way of thinking amounts to a certain way of understanding, and this way of understanding is reflected in what the Subject expresses when attempting to understand physics. As a result of the adoption of the "language-as-calculus" way of thinking, there are a number of traditional philosophical pictures to which the Subject is habituated in the foundations of physics. First, perhaps most deeply rooted, is the philosophical "model of thought," which, in essence, finds its intellectual roots in Descartes' dichotomy between matter (*res extensa*) and mind (*res cogitans*). This "model of thought" can be visualized as in Fig. 1. In this philosophical "model," reality consists of all objects, and they are beheld by the Subject, Ego or Consciousness. Something like this picture occurs for example in perceptual psychology or, say, neurophysiology, where one analyzes the process of perception in terms of light waves, pressures, etc., which act upon the sense organs of the percipient and excite certain electrical and chemical phenomena in the nervous system. Here the Subject is understood in the form of an *empirical* Subject. When the Subject is understood in this way (as in neurophysiology), it is all right.

The analogue to this picture has also been used in *philosophy*. In this context, the model of thought represented by this picture has had a paralyzing influence, because of its emphasis on the *relational* character of philosophical (semantical) thought. One is forced to assume the real existence of relations in a mysterious "metaphysical" sense. This is the case in philosophy of mathematics, as well as in philosophy of physics. In the latter, it has erected the problem of the "detached observer." In the philosophical reading of this "model of thought" (language-as-calculus), the detached Subject receives sense impressions from the objects in reality, which are organized and sorted according to the categories of pure reason, canons of induction, etc., of the philosophical "tools" into iterative complexes through which the Subject can have knowledge of reality. As far as philosophy of physics is concerned, the objects of reality are either in a metaphysical reality in general, called "physical reality," perceived (somehow) by the philosophical mind, or the objects are part of the conceptual apparatus (mind) by which sense impressions are organized. The philosophical Subject is assumed to be in a logically separated "vacuum," exhibited as a relation between Mind and Matter.

This picture, when formulated in the sense of "language-as-calculus" raises a number of problems in philosophical thinking.

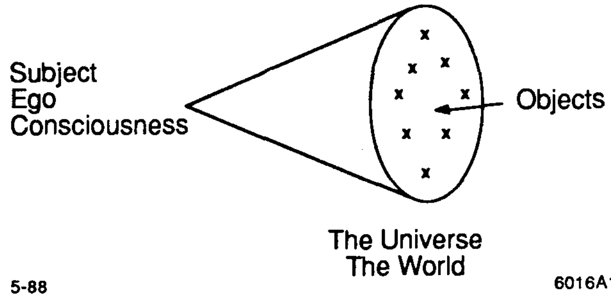

Figure 1

It provides an uneasy oscillation between the philosophical doctrines of both verification-transcendent epistemological realism and the opposite, epistemological idealism.

This is the case because the status of the philosophical Subject is unclear. On the one hand, it is present in the process of recording facts of nature; on the other hand, its influence can be neglected since reality "is there" independently of any Subject. This way of thinking implies adoption of the model of thought (language-as-calculus) exhibited in Fig. 1. As examples of physicists who have been said to have embraced a verification-transcendent realist standpoint in this sense, one can mention Einstein and Schrödinger. Above, we tried to briefly describe the realist version of this "model of thought." One can also give an idealist emphasis of the "model." As examples of physicists who have taken this latter attitude, one can mention Wigner and von Neumann. In the idealist version, the emphasis of the picture is reversed from right to left; then, reality is not so much beheld by the Subject as it is constructed by him. In this case, reality is a product of the consciousness of the Subject, and depends on him. Both interpretations, realist as well as idealist, understood along the lines of "language-as-calculus," occur in physics; thus, e.g., Barrow and Tipler point out that "(the) Many–Worlds Interpretation is often classified as a 'realist' interpretation of quantum mechanics, as opposed to the idealist Copenhagen Interpretation, which brings the observer into physics in an essential way" [33].

The problem is that, however much one assigns priority to one side of the picture or the other, the "model of thinking" (language-as-calculus) remains essentially the *same,* and the philosophical problems inherently connected with it (the objectivity of the result achieved in the practice of measuring) remain unanswered. This leads to the next inherent problem of the "model."

The picture does not say anything concerning how the two sides of the picture are connected.

This model of thought does not say anything concerning, *how,* if one has access to reality only through one's impression, the connection (relation) is set up between those expressions and what they are expressions of, or, conversely, *what* principles regulate the construction of those objects, and out of what. Whatever side of the picture is emphasized, it remains silent on this crucial point. The relation remains mysterious. As far as this problem is concerned the picture is simply not intelligible to the intelligent Subject. The problems that the two sides in the "language-as-calculus" picture gives rise to, leads to the last inherent problem of the "model":

The philosophical Subject is separated from reality, as it were, by a pane of glass, to use Wheeler's metaphor.

The philosophical Subject is a spectator or observer, watching, perhaps, shadows on the wall of a cave, as Plato formulated the problem. The Subject has no contact with reality. One could, perhaps, say that the philosophical Subject watches an internal theatre, so one may ask, "Is the Subject *unreal?*"; but this is incompatible with what we learn from the quantum principle. As Wheeler puts the point, the quantum principle "demolishes the view we once had that the universe sits safely 'out there', that we can observe what goes on in it from behind a foot-thick slab of plate glass without ourselves being involved in what goes on" [34].

This situation, again, gives rise to the following question, "Is the model of thought asymmetric with regard to different Subjects?" In order to answer this question, the model of thought is, in traditional thinking, somewhat elaborated in order to moderate its subjective aspects, as in Fig. 2.

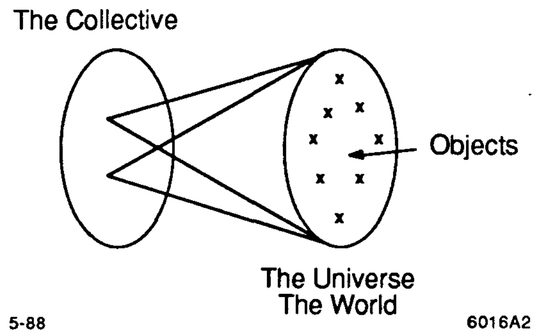

Figure 2

In such attempts, the empirical Subject is replaced by an intersubjectivity sustained between different empirical Subjects through their use of language; then, the single empirical Subject is replaced by a *collective* of empirical Subjects. To some degree, this picture, indeed, does manage to explain the objectivity of the Universe. Moreover, it achieves symmetry with respect to the observers. Despite this, there is still a crucial problem connected with this interpretation. This elaborated version of the "language-as-calculus" model of thought, still logically distinguishes, despite the intersubjective emphasis, the Subject and the Universe, whereas the Subject ought to be part of it. It still leaves the Subject *outside* the Universe.

Consequently, one again faces three (separable) parts, even if one attempts to "collectivize" the language: (1) the collective language, (2) the Universe and (3) the obscure connecting part, providing the (collective) interpretation of the Universe; also, in this case, the Subject (using a language) and the Universe are logically distinct; thus one can see that a "collectivization" of the Subject as a philosophical observer leads to the same *cul-de-sac* as in the case with the single observer. The empirical Subject ought to be a *participator* in the Universe. The quantum principle throws out the old concept of "observer" and replaces it, as far as the empirical Subject is concerned, with the new concept of "participator." That is to say, in Wheeler's formulation, "(in) some strange sense the quantum principle tells us that we are dealing with a participatory universe" [35]. The most serious defect of the "language-as-calculus" model of thought is the inability to exhibit the Subject as being a part of the Universe, and the necessity, following from this, of leaving the connection between the Subject and the Universe in obscurity.

To engage in the practice of physics in accordance with one's understanding, when based on the "language-as-calculus" model of thought, is to think in accordance with what one can call

Philosophical Separability.

The "language-as-calculus" model of thought is characterized, as we have emphasized, by three discernible parts: (1) Language and the Universe are always (logically) distinct systems. A corollary to this attitude is the doctrine of the distinction between Mind and Matter. (2) We set up the Language, say quantum mechanics (it could as well be the language of classical mechanics) to communicate among ourselves, and record (by experiments and observations) facts that we have discovered about the Universe. (3) We assign (in the fashion of logical/model-theoretic semantics) nonlinguistic items to the linguistic ones as their semantical "reference" (Frege), "denotation" (Russell) and "interpretation" (Davidson), in the hope of thereby showing their meaning and setting up the correct interpretation. This way of grasping the point of semantics is typical to the model of thought exhibiting philosophical separability. This is visualized in Fig. 3.

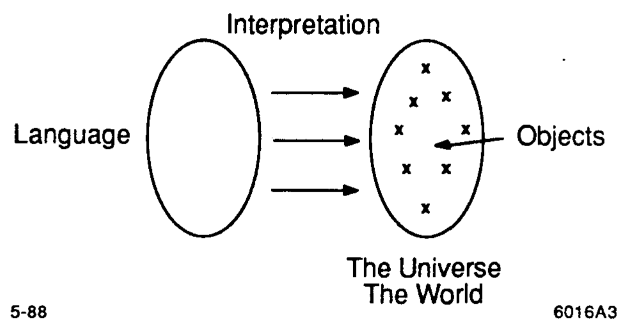

Figure 3

The outcome of the reflection on this model of thought (language as calculus) shows that there is something fundamentally wrong with this way of understanding physics. The problem focuses on the philosophical separability, which is an attitude based on illusion (as a semantical doctrine), because it makes *any* account of the connection between Language (the Subject) and the Universe obscure. According to this reading, we always interpret (a "veiled reality"?—in D'Espagnat's terminology), presupposing that we have access to anything real, actual or concrete, which we wished to make the denotation of in the Language. The problem arises because any means employed to identify that assumed real thing (of the Universe) would have an irradicable linguistic aspect (in the form of an interpretation). As examples of tacit adherence to philosophical separability, one can give the different "Quantum Realities" which are being provided as answers to the problem occurring when "(different) people looking at the same theory come up with profoundly different models of reality..." [36]. Here the assumed philosophical problem is regarded as the task of providing the (correct?) interpretation in addition (and *a posteriori*) to the quantum formalism, itself. This is not the way "theory" and "model" are understood in discrete physics. The result of the "language-as-calculus" way of thinking is that another linguistic system (the actual interpretation) interposes itself between the first linguistic system (say the quantum formalism) and its assumed field of denotation (the nonlinguistic reality). What provides the criterion that one of the formulated interpretations, indeed, is the correct one? Here, we see the problem inherently connected with this way of thinking: by virtue of *what* does one decide on the correct interpretation? Recall that we are here dealing with a relation between language and reality. Any attempt to formulate this correct interpretation requires a linguistic medium. By thinking along the lines of philosophical separability, one can *never* provide the adequacy criterion in order to decide which interpretation is the correct one. Let us look at this astonishing insight a little bit more closely.

One way of providing meaning to the various syntactical "entities" of a formal language is by modeling it in the way with which we are all familiar. The typical case would, of course, be the standard modeling of first order predicate logic. How does one proceed in this case?

- To begin with, one has a symbol, say A, for the type of individuals to which one assigns a set, which is referred to as the individual domain.
- Similarly, to each individual term, say t, one assigns an individual, say a; that is, an element of the individual domain.
- Furthermore, one assigns to each *formula* a proposition.
- Finally, one proves that if a formula is formally derivable, then the proposition which is assigned to it comes out true.

Intuitively interpreted, this means that one assigns to each formal derivation a proof of the proposition which is assigned to its end formula. This is a pattern which is followed in all kinds of modeling; most recently, in denotational semantics of programming languages, i.e., one assigns to the syntactical entities that one is dealing with certain mathematical objects and speaks of those objects as the *interpretations* of the syntactical entities. In model theory, one looks upon the interpretations in the object-oriented way in which one ordinarily deals with mathematical objects; i.e., one disregards language and handles the objects "directly," in the way one is accustomed to as a mathematician. This exhibits a mistake.

To begin with, every "object of knowledge" amounts to an *expression.* Indeed, a moment's reflection is enough to show that one is not at all dealing with these objects in a language-free way. How could one? After all, one is assigning a mathematical object to the syntactical entity by giving an *expression* for that object. One *always* uses an expression, a linguistic expression (which one, ultimately, in the last instance, must understand by virtue of one's "universal medium of communication") in order to express the object which is to serve as the interpretation of the syntactical entity. There are no "things-in-themselves" somewhere in a "linguistic vacuum."

Usually the meaning of the statement "the term **t** denotes the object **a**" is determined by three logically distinct things (in the terminology of logical semantics):

1) the meaning (use) of **t** as a term,

2) the meaning (use) of the denotation relation, and

3) the meaning (use) of the expression (interpretation) **a.**

Now, the expression **a** is, of course, *also* a term belonging to language, and it follows that in order to understand the above statement (the way it is used), the Subject must know the meaning of a statement of the form, "the expression **a** denotes the object **b,** " where **b** is another expression that denotes **a.** This last fact must be *presupposed* known when providing the actual explanation, and we are led to a regress concerning the correct interpretation. This can also be understood as exhibiting a critique against using Tarski's *correspondence theory* of truth in semantics: a true statement is a statement that is true to the facts. Here, one assumes *a priori* a certain relation between object-language and metalanguage and, again, a relation between metalanguage and metametalanguage, etc. This leads to a hierarchy of languages, the adequacy criterion of which cannot be stated. Indeed, we are led to a neverending regress concerning the meaning of the adequacy criterion. We never establish a paradigm. As Wheeler puts the point,

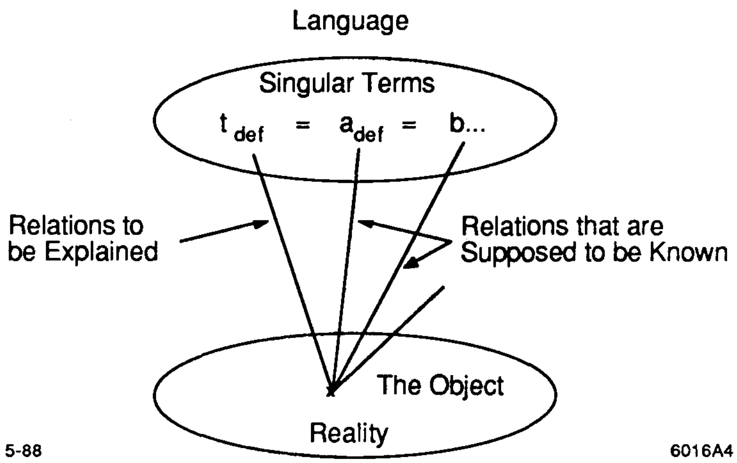

Figure 4

"How can there be an end if we ask always for foundation of foundation of foundation?" [37]. We are led to an infinite regress. The situation is pictured in Fig. 4.

It becomes logically impossible to explain *how* the object **a** could be identified at all. The source of the paradoxical result is the philosophical separation of language (the Subject) and some assumed reality, as if the separation itself would be problem-free, like an inequality $A > B$ occurring as a relation between numbers. This is usually not understood. The problem is to understand how there can be a "semantics without semantics," if we use Hintikka's terminology. By the term "semantics" in the first part of the quotation, is meant semantics in the sense of prephysics (the P–frame).

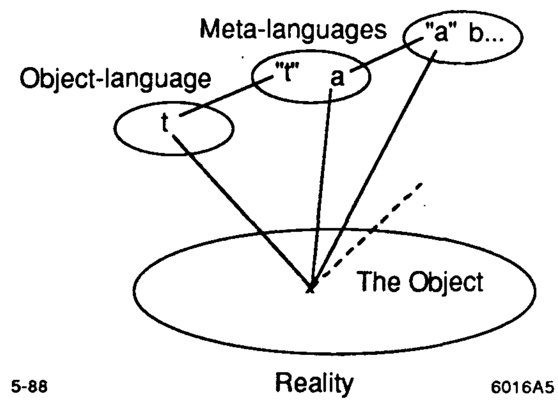

Figure 5

One might think that the problem could be avoided by introducing the distinction between object-language and metalanguage, or, what amounts to the same, the distinction between use and mention. Thus, the statement "the term **t** denotes the object **a**, " is a statement belonging to metalanguage and it is in that language that the expression **a** is *used,* while the term **t** is only *mentioned.* In order to be able to use the expression **a** in explaining the meaning of the term **t**, one must, of course, already understand the expression **a**. One must know *what* object it denotes. The result is that we have the same infinite regress, shown in Fig. 5. The distinction between object-language and metalanguage, exhibiting adoption of the "language-as-calculus" way of thinking, *itself* gives rise to the same difficulties by exhibiting adoption of philosophical separability; i.e., that one can separate a language from that (object or linguistic term) which the language treats, without being involved in the separation by having made an explicit formulation using the "universal medium of communication." This problem concerns the possibility of providing *any* interpretation. This point implies, as pointed out by M. and J. Hintikka, that "[the] impossibility of varying the interpretation of our language is an important additional reason why all model theory is impossible on the view of language as the universal medium. For a systematic variation of the representative relations between language (or at least its nonlogical vocabulary) and the world is a conceptual cornerstone of all logical semantics. Indeed, the development of logical semantics and its technical twin, model theory, has gone hand in hand with a gradual transition from the view of language as the universal medium to the view of language-as-calculus" [38]. Thus, precisely, as we pointed out in the beginning of this paper, the notion of logical semantics has made an "evolution" similar to that which the notion of function has made in mathematics; but one is not to be surprised: both logical semantics and "classical" mathematics rely essentially on philosophical separability, by treating the notion of function as a relation between arguments and value, a relation existing in a "linguistic vacuum."

One important insight to be gained by grasping the distinction between the semantical paradigms of "language-as-calculus" and "language as the universal medium" is that, in the end, philosophical problems of physics cannot be logically distinguished from philosophical problems concerning the language of physics as far as meaning is concerned. For example, the result of adopting the "language-as-calculus" way of thinking, allowing the logical distinction between object-language and metalanguage, is that we have a *diaspora* of interpretations concerning "quantum mechanical reality." This way of thinking exhibits itself in the way the problem concerning the "detached observer" in connection with the "foundations" of quantum mechanics is presented. The problem is that there is *no* universal adequacy criterion available for what is to be understood by a *correct* metatheoretical interpretation. Here, it is not a problem of providing the correct interpretation; it is the (logically impossible) task of discerning what is to count as an interpretation in the first place. This insight does not seem to have been generally appreciated in current literature on the topic.

The most important task facing philosophy of physics today seems to be to end this diaspora of "interpretations" and once more unite the understanding of the Subject concerning the practice of physics, by making the Subject grasp the practice of physics as a closed whole (a paradigm), embedded in the "life-world," i.e., in the universal medium of communication. Husserl emphasized the necessity of a paradigm, when he said "how could actual study and actual collaboration be possible, where there are so many philosophers and almost equally many philosophies? To be sure, we still have philosophical congresses. The philosophers meet but, unfortunately, not the philosophies. The philosophies lack the unity of a mental space in which they might exist for and act on one another" [39].

One simply has to accept that the Subject is being brought up in a life-world (*Lebenswelt*) by virtue of which scientific practices acquire their meaningfulness. The very point of the practice of performing experiments, for example, must incorporate practical understanding of the word "experiment" and what it implies. It must include understanding of experiments (measurements) as carrying the *semantical force of verification*. This seems to be Bohr's point, when he states that by the word *experiment* we can only mean a procedure regarding which we are "able to communicate to others what we have done and what we have learnt" [40].

To be able to engage in the practice of measuring in physics presupposes that the description of, say, an experimental setup and the result of the experiment, must be given "in plain language, suitably refined by the usual physical terminology," as Bohr formulates the point [41]. The deep insight here is Bohr's emphasis that, in order to engage in measurements in physics (in so far as understanding the point of a measurement), there is a presupposition that one is familiar with the use of a natural language as the universal medium of communication. This insight makes us understand that Bohr accepts (although never explicitly stated) the following theses:

1) Semantical relations are inaccessible.
2) The Subject cannot imagine different semantical relations.
3) Model theory is impossible.
4) Linguistic relativism is inevitable (the Subject is "trapped" in language).
5) The Subject cannot grasp Reality without linguistic interference.
6) The construction of a metalanguage is impossible.
7) Truth as correspondence is inexpressible.

The important point here is that semantical relations between language (the Subject) and reality are inexpressible (which is not to be confused with some kind of linguistic idealism). Recall that the empirical Subject is elevated to a participator in the Universe. In this sense, the Subject is always—as far as sense is concerned—embedded in a life-world (*Lebenswelt*).

Here, one is to look at the modeling in a different way—namely, think not only of what is to be interpreted as linguistic expressions, but think *also* of the interpretations which are assigned to them as linguistic expressions, expressing objects (of knowledge) in a linguistic way. Then, what appears to the model theorist as a modeling, appears—taking the attitude of semantics in the sense of "language as the universal medium"—simply as a *translation* into another language. A translation is always to be a translation into another language. Thus, we reach the insight that modeling and translation are the *same* thing within the semantic paradigm of "language as the universal medium," whether one takes an object-oriented attitude towards the interpretation or whether one looks at it linguistically. (Quine has emphasized this attitude already for a long time.) Whatever way one chooses to look at it, as modeling or as translation (within the semantical paradigm), this is certainly one way of giving meaning to the linguistic expressions of a formal language of theoretical physics. This point has been called by Hintikka the "paradox of formalization," in that language as the universal medium leads into formalism since, after excluding semantics, we retain only syntax. On the other hand, language-as-calculus also leads into formalism, since one is likely to mark those elements of language that can be reinterpreted.

One cannot conceptually separate factual knowledge and reality which the view of "language-as-calculus" assumes. This concerns *any* attempt by the Subject to formulate its understanding of the scientific physical Universe, and in addition, *any* interpretation of this understanding. This insight can be formulated as the *Transcendental Anthropic Principle* (**TAP**), which we formulated above. This principle differs from the Weak Anthropic Principle of Dicke (1957) [42], and the Strong Anthropic Principle, both in the form advocated by Carter (1974) [43] and Wheeler (1975) [44], in that these formulations all exhibit the Anthropic Principle as being concerned with *factual* knowledge and thus, essentially, being concerned with the empirical Subject.

The first to use a modern version of an Anthropic Principle seems to have been Whitrow [45], in a response to the question, "Why does space have three dimensions?" Although unable to explain why space actually has three dimensions, Whitrow argued that this feature of the Universe is not unrelated to the existence of the Subject as observer of it. Interestingly enough, the paradigm of discrete physics provides a proof that the measurable world with the richest dimensional structure consists of three dimensions, plus unobservable universal time and locally consequential time. This insight is treated in a rigorous way by McGoveran in *Foundations for a Discrete Physics* [46].

The Transcendental Anthropic Principle, essentially, tells us that it is impossible "to look at one's language from the outside and describe it, as one can do to other objects that can be specified, referred to, described, discussed and theoretized about in language," as M. and J. Hintikka formulates the point [47]. The Transcendental Anthropic Principle is another way (in a terminology perhaps more familiar to physicists) of expressing the necessity of understanding language as the universal medium of communication, when understanding that practice of physics is primary in relation to understanding physics as being concerned with objects and laws. The reason for the necessity of **TAP** is that one can use language to talk about something only if one can rely on given definite denotations (definitions), as we have emphasized above. That is, one must presuppose "a given network of meaning-relations obtaining between language and the world. Hence, one cannot meaningfully and significantly say in language what these meaning-relations are, for in any attempt to do so, one must already presuppose them" [48]. The Subject is truly a participator in the scientific Universe as far as meaning and understanding is concerned. This seems to have been emphasized by Bohr. In a discussion, as reported by Petersen, we find that Bohr is reported to have said that "...'reality' is a word in our language and that this word is no different from other words, in that we must learn to use it correctly..." [49].

Bohr is also reported by Petersen as having said that "(we) are suspended in language in such a way that we cannot say what is up and what is down" [50]. Compare this statement with the one made by Wittgenstein, when he states that "(human) beings are entangled all unknowingly in the net of language" [51]. What a striking resemblance between this statement and the one attributed to Bohr! One cannot express much clearer than this the adherence to the insight of the empirical Subject as being a participator in the scientific Universe, the insight of the *Transcendental Anthropic Principle.* This reminds one of the *hermeneutical circle*, which—in general terms—says that the Subject must always have understood in order to understand, and that the Subject nevertheless can improve this "preunderstanding" by methodological attempts to make the practice understood more meaningful, by engaging in syntax and semantics in the vocabulary of prephysics. This insight can be formulated by reflective transcendental statements like, "The Subject enters the realm of the life-world by grasping a natural language."

When the Subject learns language (the universal medium of communication), it can also perform conceptualizations; it expresses thoughts. One grasps that there are chairs, tables, trees, etc. (In Quine's terminology, "The Subject begins with 'ordinary things' and the totality of our so-called knowledge or beliefs, from the most casual matters of geography and history to the most profound laws of atomic physics or even pure mathematics and logic, is a man-made fabric which impinges on experience only along the edges.") The vocabulary is gradually expanded to include mathematical and scientific terminology. However, what is important to grasp is that immediate nature, consisting of chairs and trees (Quine's "ordinary things"), are *part* of the "background" of the Subjects preunderstanding of nature, relative to the more sophisticated language of physics. One could say that the life-world is rather like an "onion" of layers-within-layers of language, criss-crossing each other, to express the point in metaphorical terms. Expressions like "reality," "the world" or "the universe" belong to the "onion." With the words of Rorty, one can say that physics with its emphasis on objects (planets, elementary particles, quarks) and laws of nature (GTR, QED, QCD) functions like a "reflective mirror" of nature. The practice of physics becomes, essentially, like a mirror of participation in the life-world.

The upshot of this discussion ought to be the insight that there are no truths-as-facts outside logically possible experience. The scientific Universe is the totality of everything that can be the object of our experience, i.e., it is the totality of all possible experience. That is, with the words of Husserl, "[the] attempt to conceive the universe of true being as something lying outside the universe of possible consciousness, possible knowledge, possible evidence—the two being related to another merely externally by a rigid law—is nonsensical. They belong together essentially; and, as belonging together essentially, they are also concretely one, one in the only absolute concretion: transcendental subjectivity. If transcendental subjectivity is the universe of possible sense, then an outside is precisely—nonsense" [52].

Problem 5.

Prephysics amounts to a phenomenological investigation. To begin with, it amounts to grasping the insight that thought, language and Universe are one, "The transcendental ego inseparable from the processes making up his life," as Husserl puts it [53]. In the terminology of Parker–Rhodes, one can say that thought, language and Universe are *indistinguishable.* They are nonseparable ingredients of any scientific practice. Thought shows itself in the immediate acts of the Subject when attempting to solve scientific *tasks.* Thus one can say that only *acts* (Bridgman: operations) are real, actual or concrete. They exhibit performatives (= quantum principle of action). As far as the practice of physics is concerned, the most important acts are acts (operations) of *judging.* A judgment (operation) may be understood either as an act of judging (act of operating), or as *that* which is judged (the result of the operation). Likewise, an *assertion* (Ger. *Urteil*) may be understood either as an act of asserting or as that which is asserted. In its first sense, an operation is nothing but an act of knowing and, in its second sense, that which is known; that is, a piece or, more solemnly, an *object* of knowledge. The result is that judgments, operations and assertions amount to synonymous expressions, and one is to be preferred over the other only on stylistic criteria.

If one is not careful at this point, it is easy to be confused by the way the expression "operation" (judgment, assertion) is to be used in prephysics. It can, among other things, mean: (1) process of operating, (2) object obtained as the result of a process of operating and (3) operating-process as object, i.e., not understood in the sense of something "dynamic" [54]. Operations can be viewed as *processes* and differ from the resulting (constructed) object judged. The latter is a mathematical object (of knowledge) and can be *used* in the practice of physics; not so the former. In the sense one is to use the notion of "operation" in prephysics, it corresponds to form (2). This becomes evident, e.g., when contemplating on the task of engaging in syntax (formulating the R–frame). We are not interested in the "process" of what goes on in the mind of the Subject formulating the R–frame, but in the objects (of knowledge) obtained as the *result* of a "process" of operating (judging).

From the expression "operation," or "process," when understood in a representational way as, e.g., Whitehead does in *Process and Reality,* one might easily get the impression that there are actually two objects involved: the object formulated, or constructed, by virtue of the "process," and the "process" itself. This logical distinction is based on an illusion. The "process" and the object formulated by virtue of the "process" are *not* logically distinct things. They are two different ways of speaking of the same thing. To understand this, consider what must be done to construct each of them. To construct the object A, one must carry out (or perform) the construction of A. To construct the "operation" or "process" of A, one must do exactly the same. Both ways of formulating the task amount to the same: codify A. Each way of speaking has its advantages. The general insight gained is that one cannot engage in an "operation," or "process," without constructing A. Anything that needs to be proved is included. Thus, by an "operation" is to be understood a codified (in written or spoken form) object of knowledge.

Consequently, there are objects (in mathematics and physics), but only acts (operations) are actual, real or concrete. One can, perhaps, like Husserl, say that "[active] judging is not the only, but it is the original, form of judging. It is the sole form in which the supposed categorical objectivity, as such, becomes actually and properly generated. It is, in other words, the only form of judging in which the 'judgment' becomes, itself, as given originally" [55] (i.e., active judging is canonical). What the Subject has to understand, is that any assumed connection (relation) between Subject and object is *fused* in the acts of operating, performed by the Subject. One could say, as we already pointed out above, that the obscure (semantical) relation which comes last when thinking in accordance with philosophical separability (language-as-calculus), comes *first* when understanding measurement in physics according to prephysics. As far as existing "philosophical interpretations" are concerned, the reflecting Subject has to "bracket" (Husserl's terminology) the existing (language-as-calculus) convictions. Thus the Subject is faced with the task of grounding (*begründen*) the practice of measurement. This is performed by judging the *code* (R–frame and P–frame) regulating the practice of measuring. The Subject is never to deal with matters of fact, before questions of meaning have been settled, when formulating a novel paradigm.

However, an act of operating (judging, asserting) is easily read as a solipsistic act (without possibility of communication) and this, of course, is not what prephysical thinking adheres to as far as the practice of physics is concerned. This point is well formulated by Wheeler, when he states that "[what] is required in the analysis of genesis is not private judgment, but public judgment—which is to say science" [56]. But, one may ask, what is the difference between "subjective" operations (judgments) and "objective" operations (judgments)? Isn't the act of judging in both cases intrinsically connected to the judging Subject? This puzzle is dissolved by noting that by Subject is meant the *transcendental* Subject. It is this kind of transcendental Subject that Wittgenstein is addressing (despite his calling it the "metaphysical Subject"), when he writes that "(the) subject does not belong to the world; rather, it is a limit of the world" [57]. Thus the Subject engaging in prephysics, i.e., of judging the R–frame and the P–frame, is a transcendental Subject living in a life-world (*Lebenswelt*).

Here we meet an insight of crucial importance: *prephysical judgments can not be distinguished from pure realism.* The Subject is characterized by being a participator in the life-world and, thus, in the universe of intention. The Subject manifests this by exhibiting *thoughts* (*Gedanken*), and these thoughts always *mean* something to the Subject. In this sense, the Subject is always in the universe of intention, "The world and life are one" [58]. The outcome is, as Wittgenstein put it, that "... it can be seen that solipsism, when its implications are followed out strictly, coincides with pure realism. The self of solipsism shrinks to a point without extension, and there remains the reality coordinated with it" [59].

The transcendental (operating, judging) "I" (which can just as well be expressed by "you," "she," "he," etc.), coincides with the limit of the world. One easily gets the impression that there, in connection with the meaningful use of the language of physics, something psychological seems to happen, the closer inspection of which would be a purely empirical affair. This may, indeed, also be the case; but in this case, we are always dealing with the empirical Subject participating in the life-world. Now, one is not to confuse the philosophical, transcendental, Subject with the, empirical, psychological Subject; thus, the empirical Subject can be satisfied with merely noting that such and such things *must* happen (somehow) in order that meaningful use of the language of physics be possible. At that moment, the Subject steps beyond psychology and enters the sphere of transcendental reflection on the practice of physics; thus, the transcendental Subject is not totally unconnected to the empirical Subject. The transcendental Subject lays down conditions for the empirical Subject to fulfil and, as far as physics is concerned, it is the task of the transcendental Subject, by virtue of prephysics, to bring forth what these conditions are (the R–frame and the P–frame).

The code formulated can be explained in a more subjective form, i.e., a more idealistic form, emphasizing the presence of the empirical Subject (the "person program" terminology), or the explanation can take a more objective form, disregarding talk of the empirical Subject (the "theory of physics" terminology). Thus one can engage in prephysics by using a more subject-oriented way of formulation (Gefwert), as well as by using a more object-oriented way of formulation (McGoveran), by virtue of the semantical paradigm of "language as the universal medium." Both ways are imbedded in the life-world (*Lebenswelt*), being the transcendental ground for *any* understanding of the meaning of the practice of physics. The result is that whatever form one chooses to adopt, we seem to arrive at two kinds of truth in the practice of physics: (1) the transcendental notion of truth and (2) the notion of truth arrived at by virtue of the result of measurement (verification). One must be careful not to confuse these two notions of truth.

First, we have the notion of truth which is established as the *result* of a measurement (verification). This notion of truth can be called truth-as-fact. We have the analogous case in mathematical practice where a computed (proved) theorem is a proposition of the form "A is true," which is intrinsically linked with *immediate* provability (or verifiability). Wittgenstein expressed this when he said that "[the] stream of life, or the stream of the world, flows on and our propositions are so to speak verified by the present" [60]. Another way of stating this point is to say that prephysics emphasizes the importance of "do-it-yourself" (immediate) measurements in physics. In philosophy of mathematics, this way of understanding mathematical practice (analogous to prephysics) is best represented by the works of Martin–Löf.

Secondly, we have the notion of truth connected to the *validity* of a measurement in physics. This notion of truth can be called truth-as-validity. The notion of the validity of a measurement is the most fundamental notion of all, because to say that a measurement in (discrete) physics is valid, conclusive or correct, is nothing more than saying that the measurement *is* a real, or transcendentally true, measurement. It claims that a measurement is a valid measurement (a verification). Again we have the analogous case in mathematics, where we talk of a computation (proof) being valid. We can now grasp that the prephysical explanation of the notion of truth-as-fact (the analogous case in mathematics being Martin–Löf's explanation of the truth of a proposition—an expression of the form "A is true") is entirely compatible with pure realism.

Prephysics shows that the task exhibited by a question like "What objects does the world consist of?" is a question that it only makes sense to ask within the *practice* of physics regulated by a paradigm. As Wheeler says, "It tells what question it makes sense for the observer (participator) to ask" [61]. Thus, the aim with prephysics is to exhibit a *method* (the R–frame and the P–frame) in order for the Subject to be able to search and find propositions of the form "A is true," where "true" is to be understood in the sense of truth-as-fact. However, by the notion of transcendental realism is meant the philosophical insight that one can take the notion of reality for granted by virtue of the life-world. In this sense the Subject engaging in the practice of measuring in physics *already* does take the transcendental reality for granted. This means that the truth-as-fact point which one can exhibit, e.g., in the linguistic form of epistemological idealism (person program terminology), and which we, usually, exhibit in connection with physics in the linguistic form of epistemological realism (theory of physics terminology), can be understood as guiding the practice of measuring in physics, when physics is what Ryle has called "the game of exploring the world" [62]. Thus physics, understood as a "game of exploring the world," is entirely compatible with realism, if by realism is understood transcendental reality. This is already reflected in the Greek term *fysiká* ($\phi\upsilon\sigma\iota\kappa\alpha$) meaning "nature" and used by Aristotle to denote "natural science" (natural philosophy).

One is not rejecting the notion of "reality to be discovered" when engaging in the practice of (discrete) physics. As far as this point is concerned, there is no difference what paradigm of physics (continuum or discrete) the Subject adheres to. Whatever paradigm of physics the Subject uses, the transcendental reality is always presupposed. Maybe one can, like Prawitz, say that "(the transcendental) world is not there independently of us, but given that we are here, the world is also there waiting to be discovered" [63]. Indeed, to not adhere to this insight would amount to a genuinely irrational standpoint: to perform measurements without understanding the very *meaning* (point) of the practice of measuring in physics. Practice of physics would amount to an irrational practice for the Subject despite the fact that the Subject performs this very practice; a paradoxical situation. The Subject would be like a savage looking at an artifact (like a computer) not grasping what it is and what it is designed to do (despite being engaged in pushing certain buttons on the keyboard).

Another way of putting it, is to say that in such a case the Subject would not understand *what* to do when encountering such an artifact. Thus, the actual, or immediate, practice of measuring, just by being performed successfully, exhibits that transcendental reality is being adhered to. The actual practice of physics itself, just in the very performance of it (in whatever paradigm), *shows* that transcendental reality is being adhered to. Doubt at this level amounts to doubting the very meaning of measurements performed in physics. For a physicist it amounts to doubting the very meaning of the measurement he is currently performing. For a mathematician it would amount to doubting the very meaning of the computation (proof) he is currently performing. For the theoretical physicist it would amount to doubting the very meaning of the theory he is currently formulating or the calculation he is currently performing by virtue of the theory. For the ordinary human being it would amount to doubting the very meaning of the sentence he is currently uttering with the intention of conveying a certain point. One can say that practice is speaking for itself. In this sense the participating Subject is *always* a transcendental realist *au fond.*

III

Problem 6.

For the Subject to engage in the practice of physics presupposes practical competence to perform certain *tasks* in physics. One cannot exhibit any methodological rules for how to achieve practical competence. Practical competence does not *primarily* amount to verbal explanation (although one usually needs verbal explanations, i.e., semantics, in order to understand how to perform the task set out to achieve). To exhibit practical understanding is to exhibit *competence* to actually perform experiments and computations on request. This practical competence cannot be substituted for theoretical (descriptive) understanding of practice. The starting point must be the practical capacity to know how to perform measurements and computations in physics. One can list certain informal (heuristic) conditions that the Subject has to meet in order to achieve practical understanding (the E–frame):

(1) Agreement of cooperative communications
- commonly defined terms as fundamental
- fundamental versus derived terms
- agreement of pertinence

(2) Agreement of intent

(3) Agreement of observations

(4) Agreement of explicit assumptions

(5) The Razor

- agreement of minimal generality
- agreement of elegance
- agreement of parsimony

As we stated above, practical understanding of physics amounts to the practical competence to perform certain tasks in physics when requested to do so. These tasks are those which one normally would say correspond to the tasks of a trained "experimentalist" in the laboratory. However, practical understanding occurs also in what we call "theoretical physics" (including mathematics and computing/computer science). Here the practical understanding shows itself in the competence to formulate, explain and calculate, with theories of physics in the sense of a trained "theorist." The point of courses, examinations and laboratory training (including mathematics and computer/computing science), is precisely to convey the skill exhibiting itself as practical understanding of physics. In Kuhn's terminology, the practice of the "experimentalist" and the "theorist" belong to *normal science.*

As was realized a long time ago, there is no theoretical method by virtue of which the practical competence can be achieved. This is reflected in the term *heuristic (ars inveniendi),* by which is (and was in classical Greek) meant the methods and rules of discovery and invention. Important sources when investigating the heuristic method are provided by Euclid, Pappus (who has very interesting comments on the topic), Descartes and Leibnitz. In the last century the topic of heuristic was investigated by the philosopher Bernhard Bolzano in his *Theory of Science (Wissenshaftslehre)* of 1837. The modern investigations *par excellence* have been provided by Polya in his *Mathematics and Plausible Reasoning* and by Lakatos in *Proofs and Refutations.*

Now, it is clear that the informal conditions for achieving reflective equilibrium in the practice of physics can be understood as a certain kind of heuristic advice. To conclude: practical understanding (the E–frame) consists no more in the ability to state nor to describe verbally how a measurement, or the expressions of quantum mechanics are to be used, than the ability to drive a car consists in the capacity to describe how car driving is done. A similar example would be to stress that the Subject does not learn to talk by being told theoretically (physiologically) what happens when a person talks. To assume this (which seems to happen too often in philosophical or foundational discourse) is to become victim to what Ryle has called "the intellectualist legend," i.e., the illusion that intelligent performance involves explicit observance of rules. This point is reflected in the criticism of the possibility to formulate any "logic of induction" that prephysics exhibits. This insight was also reached by Einstein in his lecture *On The Method of Theoretical Physics,* where he states that "any attempts to derive logically the concepts and laws of mechanics from the ultimate data of experience is doomed to failure. There is no inductive method that can lead to the fundamental concepts or principles. The truly creative principle of theoretical physics is mathematical construction" [64].

Problem 7.

Wheeler has suggested that human communication is an essential part of the formulation of the laws of physics. This requirement amounts to that of point (1) of the modeling methodology of prephysics: agreement of cooperative communications. Both require adherence to the view of language as the universal medium of communication. We shall now investigate this specific question in more detail. Above we said that the practice of physics (the E–frame) presupposes natural language in the sense of a "universal medium" of communication, in Hintikka's vocabulary. Here we meet again an important point when understanding the practice of physics: the nonseparability of *rule* and *act*. This is exhibited in the very etymology of the word "practice."

Since the Subject participates in the Universe, every *meaningful* operation (judgment, assertion) exhibits a rule. For example, if we stick to physics, the practice of measuring a fact exhibits a number of rules (operations), making it possible to *repeat* the result of a measurement (operation). To make the point more precise: every scientific operation must be a *repeatable operation* if it exhibits a factual claim. This shows the criterion of objectivity relative to the practice of physics: the possibility of repetition. As Wittgenstein formulates this point, "The limit of language shows itself in the impossibility of describing the fact that corresponds to a sentence... without repeating that very sentence" [65]. This concerns operations (speech-acts) of informal speech and writing (e.g., this article), as well as operations (performatives) in the practice of physics.

It is this point which makes the Universe, when exhibited in the practice of physics, a participatory Universe. This has not always been understood. The way laws of nature are being understood, e.g., exhibits this misconception clearly. Empirical "law-like" statements such as "The sun rises every morning" and "If a stone is dropped it falls to the ground" differs, so the story goes, from observational statements such as "The car is black," in that the latter statement, but *not* the former ones can be conclusively verified by observation. The former "law-like statements" can only be confirmed to a *high degree,* or can only be given a high degree of inductive support, according to conventional wisdom. They cannot be conclusively verified. They nevertheless express a definite empirical content, but this content can, in principle, only be known to an (unlimitedly) high degree. Another way of stating this conventional wisdom, is to say that laws of nature have *empirical* content which, on good grounds, we believe in (like Newton's laws, or, say, Einstein's General Theory of Relativity), but which in principle cannot be known with certainty. This inherited way of thinking is, it seems, very common. It is not always explicitly stated, but *shows* itself in what one is being taught in physics.

However, it contains a conceptual confusion. To say that a statement has a definite meaning or a definite content, which in principle cannot become complete knowledge, *is a paradoxical statement when the practice of physics is understood as being participation in the practice.* It is a paradoxical statement because to say that a statement has a definite meaning *is* to say that it expresses possible knowledge. The statement is meaningful since otherwise one could not grasp the very point of it. The Subject uses this statement in order to convey a certain *point.* When conventional wisdom states that the statement has a definite meaning which cannot become complete meaning, it assumes that the Subject can somehow "separate" itself from the universal medium of language.

The above way of thinking about laws of nature (language as calculus) is mistaken. Not only do we know many laws like the ones above, or more complicated ones, like Newton's first law of motion, "Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change state by forces impressed upon it," to a high degree—as conventional wisdom requires; the Subject actually knows these laws with *certainty.* As Wittgenstein put it, "All testing, all confirmation and disconfirmation of a hypothesis takes place already within a system. This system is not a more or less arbitrary and doubtful point of departure for all our arguments; no, it belongs to the essence of what we call an argument. The system is not so much the point of departure, as the element in which arguments have their life" [66].

The error is to a great extent based on the following misconception: a law of nature is assumed to have the character of a universal statement (or proposition). Usually by a law of nature is thus meant *empirical generalizations.* As examples of such laws, one can give the velocity of light or the gravitational constant. The meaning of a law-like statement is determined in terms of the meaning of its instances. The correctness of the law consists in the correctness of all its instances. This, however, is an illusion, because if it were correct, the Subject could not know the meaning of a single instance. A law of nature is not a universal statement (proposition), but a *law,* and a law is a rule or principle, something that the Subject follows; thus, a law of nature has to be something else than what one usually finds in books of physics (and philosophy). A law of nature does not express an empirical fact in the sense of something that the Subject can verify by observation or by engaging in an experiment. No, our basic laws of nature are what makes it *logically possible to make empirical observations at all.*

A law of nature belongs to the R–frame, formulated in syntax, and the P–frame, explained in semantics, in the terminology of prephysics. A law of nature (in contrast to, for example, a statistical law) is something that the Subject can only understand, and this knowledge of meaning is logically prior to the knowledge of an instance of it. Einstein seems to use the notion of "hypothesis" in this way when he writes that "[this] stipulation contains a further physical hypothesis,... . It has been assumed that these clocks *go at the same rate* if they are of identical construction" [67]. This statement cannot be conclusively verified within the practice of the Special Theory of Relativity. On the contrary, this statement makes it possible to formulate statements which can be verified in the practice of physics using the Special Theory of Relativity. Such a statement belongs to the E–frame in the method of prephysics; i.e., it is a statement concerning which one can only have practical understanding, providing one is to grasp the point of the Special Theory of Relativity.

To understand a law of nature in this sense amounts to being able to *use* it; thus, for example, expressions of natural language (when used) can be understood as exhibiting natural laws, in this sense, The rules of mathematics (the R–frame) and the rules of explanation (the P–frame) occurring in the practice of physics are also rules of nature in this sense. The laws of nature that we *do* follow in practice are genuine laws of nature. The Subject does not follow them *because* they are the genuine laws. That there are genuine laws of nature means that they are the ones that the Subject, usually implicitly, follows in practical computations and measurements. These laws of nature amount to *language-games* exhibiting the *ineffability of semantics* (in the language-as-calculus sense) in Hintikka's terminology.

By the notion of a law of nature, we thus mean the necessary rules that the Subject uses in practices of mathematics and physics: the paradigm. This is in agreement with Husserl's distinction between laws of nature and laws of *essence.* In prephysics we reverse this point. What we have called laws of nature are what Husserl calls laws of essence. It is these laws of nature (essence) which make it possible for the Subject to convey the possibilities of the objects of knowledge to combine with each other. These laws of nature (essence) are, and can be established as, self-evident. These laws of nature regulate what the Subject can or cannot practically understand. To engage in the practice of physics, by using these necessary laws of nature is *not,* as we said above, based on empirical generalizations. These laws are, as Wittgenstein put it, even more inexorable than the laws that empiricists (scientists) usually call laws of nature [68]. Another way of expressing the notion of a law of nature in the sense used here, is to call it a *law of thought* (law of practice). However, this does not mean that such a law is some kind of psychological law. As Wittgenstein said in one of his lectures in 1939, "The question is whether we should say we cannot think except according to them, that is, whether they are psychological laws—or, as Frege thought, laws of nature. He compared them with laws of natural science (physics), which we must obey in order to think correctly. I want to say they are neither" [69].

These laws are neither psychological laws nor are they laws of nature in the factual sense emphasized by empirical thought when read according to the "language-as-calculus" idea. A law of nature is, consequently, a law for what the Subject does when it engages in experiments and observations.
The basic system of rules (the paradigm) which the Subject (implicitly and explicitly) *uses* in the practice of physics *are* true laws of nature. They are constitutive of the meaning of the notion of empirical truth (truth-as-fact). From an empirical point of view, our basic laws of nature are absolute. The laws of nature are normative (and descriptive) rules regulating the practice of physics, making the practice what it is: a paradigm of physics.

What we have said here, of course, does sound strange for someone who is thinking as if the Subject were an observer of the Universe, being conceptually *outside* the Universe. This way of thinking is characteristic of the empirical way of thinking, and is rooted in the mechanistic philosophy of the seventeenth century. It is this view, which lies behind the conception of a law of nature, that we have criticized. This view suggests that there is some conceptually neutral way of observing objects and events in nature, independently of the laws of nature (the language-as-calculus view). It is not generally understood that the Subject is always conceptually *in* the Universe, and therefore always follows laws of nature when performing practices of physics. In general, however, one does not understand that this is the case. This attitude shows itself clearly, e.g., in talk of the "Big Bang" when the current "laws of nature" is said to have been formed (Wheeler: "mutability of laws of physics"). It is not generally grasped that talk of the "Big Bang" is a *metaphorical* way of expressing one's current expert knowledge of physics (analogous to the metaphorical way of talking of the Law of Excluded Middle in logic and mathematics). Thus one can say that a law of nature, as read in prephysics, does not amount to a *hypothesis* about what happens in some occult metaphysical reality, which is what the criticized view would amount to in the end. A paradigm is not a hypothesis in the sense familiar from theoretical physics.

In order to be able to participate in tasks of computing and measuring in physics, the Subject must be within a certain conceptual system or system of laws (the R–frame and the P–frame). To observe and describe the Universe in its variety, outside any conceptual system (paradigm) would be, as Einstein put it, like breathing in a vacuum. This is exhibited, for example, when Einstein showed that it is possible to use local, consequential time to *replace* the concept of Newton's absolute space and time. Recall that the concept of the homogeneity and isotropy of space used by Einstein (because of the need for boundary conditions in setting up a general relativistic cosmology), to analyze the meaning of distant simultaneity in the presence of a limiting signal velocity is, in fact, very close to Newton's absolute space and time. It was reflection on the *meaning* of the notion of time, space, simultaneity, etc., that led to the Special Theory of Relativity. What Einstein realized was that these notions have no absolute meaning independently of *what the Subject does* when observing and measuring in physics.

However, when it measures a fact within a conceptual system (paradigm), the Subject cannot relativize to *that* particular system without moving into another conceptual system. As Wittgenstein put it, "We are confronted here by a kind of theory of linguistic relativity. (And the analogy is not accidental)" [70]. Indeed, the analogy with Einstein's Special Theory of Relativity is not accidental. Recall that Einstein *motivated* his theory by discussing the ways in which certain propositions (ascriptions of simultaneity and time) can or cannot be verified. It is only possible to verify a proposition relative to a certain conceptual system (paradigm). The system (paradigm) cannot itself be verified. It is the conceptual framework relative to which verification is possible.

It is only from a logical point of view that certain laws of nature are relative (conventional). No paradigm is absolute from a *logical* point of view. In this sense the Universe is not deterministic. On the other hand, everything in nature must be understood in some conceptual system. In this sense the Universe is deterministic. This important point was emphasized by Poincaré and Einstein but has been misunderstood in the tradition exhibiting factual thinking (language-as-calculus).

In prephysics the conceptual system (paradigm) consists of the E–frame, the R–frame and the P–frame. The conceptual system (the paradigm) cannot be proven true (in the sense of truth-as-fact), it can only be understood. Wittgenstein formulated this insight by saying that "[the] thing that's so difficult to understand can be expressed like this. *As long as* we remain in the province of the true-false games a change in grammar can only lead us from *one* such game to another, and never from something true to something false" [71].
Problem 8.

The next point to be investigated is the requirement exhibited in point (2) of the modeling methodology: the requirement concerning agreement of intent. This amounts, when put in general terms, to the requirement concerning explicit specification regarding the *aim* of prephysics. In general terms, *the intent to engage in a prephysical activity can only be motivated by a requirement of making the practice of physics more meaningful* as a result of conceptual and practical (computational) problems existing in the current (continuum) paradigm of physics. When put into action, this is done in the form of heuristic advice.

Above, we gave certain heuristic conditions. We can now specify the requirement concerning agreement of intent when engaging in the "paradigm shift," in Kuhn's terminology as follows: one has to (1) *understand* the task (problem) to be met by syntax and semantics, (2) carry out the task of formulating the R–frame in syntax and (3) find the connection between the data (the E–frame) and the formulated R–frame in semantics (the P–frame). When the Subject has learned to put the P–frame into intelligent use, the reflective equilibrium is restored; we have achieved "normal science" in Kuhn's terminology. This closes the practice as far as meaning and understanding is concerned; the Subject practices "normal physics," i.e., we have agreement of intent when performing physics as a practice.

When the Subject has grasped how to engage in the practice of physics according to the novel paradigm, but *also* has explicit knowledge of the R–frame and the P–frame, he has what Gefwert calls *theoretical* understanding of the practice: an explicit method to use in order to find answers when measuring in physics. Thus, it is by virtue of theoretical understanding that the "counter paradigm" of Noyes is to be understood. In Noyes' formulation it reads, "Any *elementary event,* under circumstances which it is the task of the experimental physicist to investigate, can lead to the firing of a counter" [72]. However, there is an important point missing in Noyes' formulation.

As we said above, when the Subject has achieved theoretical understanding it has a method (explicit knowledge of the paradigm) for finding answers in the practice of physics. This is reflected in the etymology of the word *method,* which is derived from the Greek *meta* ($\mu\epsilon\tau\alpha$), meaning "after," and *odos* ($o\delta o\varsigma$), meaning "way." We have emphasized throughout that one is to grasp the practice of physics as being primary, in distinction to physics as being concerned with objects and laws. This leads to the requirement to distinguish, in Dummett's terminology, between *implicit* knowledge of meaning and *explicit* knowledge of meaning, of the counter paradigm [73]. The distinction between implicit, and explicit, understanding, or knowledge of meaning, is reflected in the "circumstances" which it is the task of the experimental physicist to investigate. These "circumstances" can be understood as referring (1) to the theories (person programs) used in everyday practice of theoretical physics, and (2) it can refer to explicit understanding of the paradigm itself: the ordering operator calculus and the (applied) Intuitionistic Theory of Types (Sets), coding the inference rules applied in the practice of theoretical physics within the paradigm. This distinction plays an important role in the modeling methodology itself, in that the practice presupposed (in prephysics exhibited as the E–frame) in the end *must* be explained as implicit knowledge of meaning of this practice. This is reflected in the necessity of the Transcendental Anthropic Principle.

The distinction between implicit and explicit knowledge of meaning is well formulated by Prawitz, when he states that "[knowledge] is explicit when the person can state what he knows, i.e., when he can assert some sentences that express the content of his knowledge; and then, of course, it is implied that he knows the meaning of the sentences that he asserts. To explain all knowledge of meaning as explicit knowledge would thus necessarily be circular, since any such explanation presupposes what it is to know the meaning of some sentences. Dummett's important conclusion is that knowledge of meaning has in the end to be explained as implicit knowledge, i.e., in terms of some practical ability, which of course must be some ability with respect to the use of language" [74]. This insight is also reflected in prephysics as exhibited by the modeling methodology and Noyes' counter paradigm.

To have explicit understanding (or knowledge of meaning) of the ordering operator calculus, amounts to having theoretical understanding of the practice of physics when coupled with the rules exhibited by the P–frame. The rules that determine the practice to what it is (the R–frame and the P–frame), constitute the theoretical, or objective, side of the Subject's actual knowledge of the practice of physics. When reflective equilibrium is achieved, i.e., when the method is being used (in an implicit way) in the immediate practice of physics (the E–frame), the paradigm constitutes the objective ingredient of the practice. Thus, one can say that the paradigm, as such and when applied, is naturally understood as the theory of knowledge of the practice of physics; that is, of the *demonstrative* knowledge of the practice of physics; prephysics amounts to exhibiting what Aristotle called *epistéme apodeiktiké* ($\epsilon\pi\iota\sigma\tau\eta\mu\eta$ $\alpha\pi o\delta\epsilon\iota\kappa\tau\iota\kappa\eta$), for the practice of physics.

Problem 9.

We are now able to investigate the prephysical condition put forward in point (3): agreement of observation. As far as practices like mathematics and physics are concerned, the outcome of the previous discussion is the insight that the Subject does not investigate any assumed relation between, for example, the language of physics and reality, even if the Subject assumes being involved in such an investigation. This insight seems to have been understood by Bohr. According to Petersen, who was Bohr's long-time assistant, Bohr once declared, when asked whether the quantum mechanical algorithm could be considered as somehow mirroring an underlying quantum reality, "There is no quantum world. There is only an abstract quantum physical description. It is wrong to think that the task of physics is to find out how nature *is*. Physics concerns what we can say about nature" [75].

This comment has sometimes been interpreted as expressing that Bohr is adhering to some kind of philosophical idealism (or instrumentalism), when understood in the sense of "language as calculus." This is a mistaken way of reading this quotation. What Bohr is saying is that physics as a practice is more fundamental than physics as concerned with factual laws and factual objects. Bohr adheres to the view of "language as the universal medium." This point has not been generally grasped. Physics as a practice aims at measuring facts. It aims at the notion of truth-as-fact; i.e., to say something "about" nature. To *say* (judge, assert) something about nature in this sense, is to judge some expression with a certain semantical *force,* in Frege's terminology. By performing this task, the Subject exhibits his participation in the practice, and thus implicit adoption of an R–frame and P–frame. By participating in the practice, the Subject also participates in transcendental reality. Adherence to transcendental reality is a presupposition in order for the practice of measuring to make sense.

What the Subject can say "about" nature is to be understood as expressing the insight that in the end, what is to be counted is what the Subject can *immediately* express by the measurement. This point is intrinsically connected with immediately *observing* the result of a measurement. To observe, in the practice of physics, is always to be within a conceptual framework (the R–frame and the P–frame). One can express this point by saying that observation is always *paradigm-laden.* Only a Subject "sees," not the eyes of the Subject. Cameras and eye balls do not see; they are conceptually blind. To "see" or "observe," is to be understood as exhibiting the semantical *force* of the R–frame and the P–frame; thus "observation" or "seeing" is to be grasped in the metaphorical sense, where the words "see" and "observe" are synonymous with "observing" (understanding). In the practice of physics, we always "see" in the sense of "seeing as" or "observing as." All seeing is "seeing as." If a Subject sees something at all, it must look like something. Another way of stating this is to say that "observations" are always understood within a practice of physics and thus, implicitly exhibiting an R–frame and a P–frame, when grasped in the sense of "language as the universal medium."

We shall now attempt to exhibit this insight by formulating what is meant by saying that acts, or events, are to be understood as being *immediate.* Assume that a Subject is looking at, or observing, two rods *a* and *b* lying in front of him. Assume, furthermore, that the two rods are placed at some distance from each other in such a way that the Subject is unable to see which of them is the longer one. Furthermore, assume that the Subject adopts the following working hypothesis, "rod *a* is longer than rod *b.* " Let us call this statement A. Look at A as a kind of primitive scientific theory. When this formulation has been performed (past tense!), the expression A is of the form "A is a proposition." The proposition formed exhibits a certain task: the task to *become* verified, i.e., to be established in the form "A is true (fact)." Here we distinguish *two* components in the expression A: *proposition* and *force.* The (semantical) force can exhibit a number of forms. One can express by ⊢A, !A and ?A, the assertion, the command and the question, respectively, the proposition-component of which is A; thus the proposition A, "rod *a* is longer than rod *b*" first has the force of a question: ?A (within the paradigm).

In order to answer the question, one has to engage in a measurement. Thus, the aim becomes to engage in falsifying or corroborating this statement, i.e., to affirm the proposition A. In order to perform this task, the Subject is to employ a certain *method*: the paradigm of the practice of discrete physics. Note that as the situation has been described, the statement A purports to express *what* to do in order to exhibit this statement as being a *fact* (truth-as-fact). In order to *find out,* in accordance with the task at hand, if the rod *a* is longer than rod *b,* the Subject has to engage in performing a measurement. The Subject can be understood as engaging in a task of searching and finding a fact, by virtue of the measurement. The Subject has to engage in a practice of searching and finding, verifying, or measuring, the factual truth of the proposition A. In order to achieve this, assume that the Subject brings the two rods *a* and *b* together, and that it looks like this:

The result of the experiment performed exhibits the force ⊢A of the proposition. As a result of having performed this immediate experiment, the Subject is justified in asserting !A, i.e., that "the result of the experiment shows that rod *a* is longer than rod *b.* " By bringing the two rods *a* and *b* together, the Subject exhibits (implicit) knowledge of *what* to do in order to solve the task A. One can also say that by bringing the two rods *a* and *b* together, the Subject exhibits practical competence (understanding) in order to solve the task at hand. When the Subject,

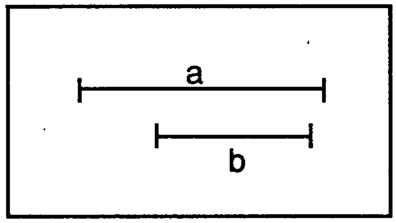

Figure 6

then, as a result of having performed the measurement, asserts that rod *a* is longer than rod *b*, this shows that the Subject knows *that* rod *a* is longer than rod *b*. The general question of what it is to know the meaning of A, can now be given the informative answer, that it is to know what counts as a direct (immediate) corroboration of it.

The statement A above is of the kind that one *understands* rather than corroborates. Someone who was looking at the two rods lying close to each other, but who did not assent to the statement ⊢A, would *not* be in need of making another observation (experiment). He would rather need an explanation of the meaning of the experiment. One could say about such a person that he does not know what it *means*, in general, for a certain rod to be longer than another. One could characterize the situation by saying that (1) the Subject does not know what to look for, (2) the Subject is not able to recognize the result of the measurement, (3) the Subject does not know when the task has been solved, and (4) the Subject does not know when the question has been settled. To know the meaning (point) of the measurement in the example above, amounts to grasping the proposition A as a *problem* (Kolmogorov), *expectation* (Heyting), or *intention* (Husserl). When the Subject has brought *a* and *b* together, he has corroborated (verified) A to be a fact. The Subject knows *how* to verify A and he knows *that* this fact obtains. The Subject *sees that* A is a fact. Note that "seeing that" is always connected to a sentential clause. The Subject sees that A is a fact, where A *always* stands for a complete *sentence*: the vehicle of thought (cf., Frege's *Context Principle*: Never to ask for the meaning of a word in isolation, but only in the context of a proposition). In Bohr's terminology, the Subject grasps that A is a *phenomenon*.

This leads to the interesting insight that both *verification* and *corroboration/falsification* belong to immediate practice of physics. Relative to the *paradigm* (the R–frame and the P–frame), the Subject can be said to *verify* a proposition A to be a fact. Verification is thus connected to explicit knowledge of meaning. However, when performing a measurement *within* the paradigm, the Subject can—relative to that practice—be said to corroborate (or falsify) the proposition A. Thus, one can say that when the Subject corroborates (or falsifies) a proposition A, he has *implicit* knowledge of the paradigm used. Corroboration (or falsification) always takes place *within* the paradigm applied. Consequently, it is a question of relative to *what* [paradigm or theory (person program)] a fact A is grasped when being established, which determines whether it is verified or corroborated (or falsified). Note that in either case, the practice always *terminates* when the proposition A is found to be true (truth-as-fact). Another way of formulating this insight is, in Dummett's terminology, to say that whether one verifies or corroborates (falsifies) is dependent on whether the Subject has explicit knowledge of meaning (verification) or whether it has implicit knowledge of meaning (corroboration). In speech act philosophy (Austin's terminology) this is expressed by saying that implicit knowledge of meaning, when corroborating a fact, exhibits an *elocutionary force*; it is not an explicit part of what one corroborates, but is implicit in corroborating A to be a fact.

The notion of *verification* can be replaced by *solution, fulfillment, realization, winning strategy* or *measurement.* When the Subject knows that the statement A is a fact, he also understands the configuration of *a* and *b.* One cannot understand the proposition as being true (expressing a fact) without *having performed* the act (operation) of corroboration (verification). Thus, one can understand Einstein's point that a concept does not exist for the physicist until he has the possibility of discovering whether or not it is fulfilled in an actual case. This is also, precisely, what Bohr emphasizes when he stresses that no elementary phenomena (proposition) is a phenomena until it is an observed phenomena (fact). To understand A (to know how to verify it) and to know that it is true (truth-as-fact) amounts to the same thing; the meaning of the configuration is the meaning of the truth (as fact) of the proposition A. To know *how* A is true (a fact) and to know *that* A is true (a fact) amounts to the same immediate practice.

The meaning of the configuration (exhibiting practical understanding *how* to verify) and the meaning of A as true (expressing a fact) are logically nonseparable. Once a certain result A of a measurement has been observed (or found), i.e., A is a phenomenon (Bohr), then it does not take another measurement, or observation, to know *that* A is the result of the measurement. This is the case, because the result A is the result of the measurement, and to observe it *is* to observe it as such. The analogous case in mathematics amounts to grasping that one cannot prove that a proof is a proof; this can only be understood. When the Subject reaches the words Q.E.D. at the end of a proof of a theorem, one is supposed to have understood that it is a proof of the theorem in question. To identify the result is to understand it as a result (phenomenon) of the experiment. In this case the Subject is entitled to judge (assert) that A (1) is a *fact,* (2) is *true,* and (3) is a *phenomenon* (Bohr). The Subject is truly a participator in the practice of physics in the sense required by Wheeler.

IV

According to Hintikka "...as we cannot have any knowledge of things-in-themselves but only in a framework of knowledge-seeking activities, the references of our expressions cannot be given independently of those activities either" [76]. By a "knowledge-seeking activity" in physics is to be understood the immediate practice of measuring an object (of knowledge) to become a fact. This always takes place within a certain paradigm. The "knowledge-seeking activity" consists of two parts: (1) the activity of formulating a theory of physics (a person program) where the Subject is *searching* for knowledge of fact, and (2) the activity of *finding* a fact, i.e., the activity of engaging in measuring a proposition to be true (truth-as-fact). Thus, one can grasp that a practice of physics performed within a paradigm (either the continuum and discretum) consists of the immediate activities of searching and finding facts.

The discussion above leads inevitably to the insight that the primary task of prephysics is not affirmation or denial of the existence of some metaphysical objects, be they partially "veiled" or not (to use D'Espagnat's terminology) when understood in the sense of "language-as-calculus." The task of prephysics is to formulate and explain the *paradigm* of physics, consisting *both* of exhibiting the syntactical part of the paradigm as well as the semantical part of the paradigm. This is *not* to say that there are no objects; there certainly are, but the objects of physics are not metaphysical "veiled" objects (D'Espagnat) or some kind of "Ding–an–Sich" (Kant). The objects of physics are objects of knowledge, judged by the Subject to have meaning and understood by virtue of measurements. In other words, Physics is to be understood, primarily, in the sense of "language as the universal medium." However, this is not enough. Physics, when understood as an immediate practice of searching and finding facts should also be understood as exhibiting a *finite discrete* structure.

The reason for this is the general requirement that once a fact of physics is established, within the paradigm of discrete physics, one should always, on request, be able to exhibit an effective computable method (theory, person program) showing how to find the fact. This can be seen as a generalization of the constructivist tenet in mathematics, "There is an x such that $P(x)$," *means* we can explicitly produce an x such that $P(x)$. If the solution to the task (problem) at hand depends on some parameters, we must be able to produce the solution explicitly by some *algorithm* (rule) when given values of the parameters. That is, discrete physics requires that "for every x there is an y such that $P(x, y)$," where x is the measured *result* (truth-as-fact) and y is the explicit theory (person program) being the *method* for finding x. Thus, every theory of physics (person program), when formulated within the paradigm of discrete physics (the R–frame and the P–frame), implicitly exhibits a computer program (due to adoption of the notion of *function* occurring in constructive mathematics).

From what has been said so far, it is not difficult to grasp the relevance of constructive mathematics (applied in discrete physics) for computer programming. According to Nordström [77]:

1. The notions of *computation* and *method,* basic for constructive mathematics, is exactly the same as in Computer Science; it is a method which when applied to an argument of the right kind will output something of the right kind. The function concept in classical mathematics (a subset of a cartesian product with certain properties) is not what programmers use.⋆

2. From a constructive proof of a proposition it is possible to construct a program which computes relevant information from the proof. For instance, a proof of an existential proposition $\exists x.P(x)$ will yield a program which computes an object a which has the desired property $P(a)$.

This point can now be extended to the practice of theoretical physics when understood within the paradigm of discrete physics. Here, one is to apply constructive mathematics when formulating theories of physics (person programs). This amounts to understanding discrete theories of physics as, implicitly, exhibiting high-level programming languages. The practice of physics, if founded *(fundiert)* on the paradigm of discrete physics, is effectively computable. It does not employ any kind of *Principle of Omniscience,* in Bishop's terminology, which lies at the root of most of the unconstructivities of classical mathematics. As an example of the principle of omniscience, one can provide the following: if $\{n_k\}$ is a sequence of integers, then either $n_k = 0$ for some k or $n_k \neq 0$ for all k. This is called the *Limited Principle of Omniscience* (**LPO**), which states that:

$$\forall f \in \mathbf{N}^N[(\exists n f(n) \neq 0) \vee (\forall n f(n) = 0)] \ .$$

According to the absolute conception of truth (exhibited in the language-as-calculus semantics), this disjunction is true: the right side is true if every n has the quality f; if this is not

⋆ This last statement of Nordström's is debatable; consider a program function which encodes a look-up table or a sort-merge algorithm.

the case, the left side of the disjunction is true. Assume that the right side is false. When the Subject starts the routine of searching and finding along the natural numbers, it sooner or later bumps into such an n for which f is not valid. The Subject has found a number which validates the existential proposition on the left-hand side of the disjunction above. One of the best known, so far undecidable, problems in mathematics is the *Riemann hypothesis* which can be formulated as follows: $c = 0.5000 \ldots$. Now, let $f(n)$, where the expression $c's\ n + 2'nd$ decimal is 0. The expression c is given in such a manner, that one can count its decimals indefinitely. Now, $\forall n f(n) = 0$ states that $c = 1/2$ and $\exists n f(n) \neq 0$ states that $c > 1/2$. According to the constructivist meaning of the logical operations LPO is not justified, even though we saw that it is classically true. Because we have not proved that $c = 1/2$ or $c > 1/2$, and, furthermore, since it is a consequence of LPO, the Subject has not proved LPO. Since we assume that we can always find undecidable (so far) mathematical problems of the form, "is every decimal 0 or is some decimal different from 0," we believe that LPO cannot be proven in a constructivist way. Classically, LPO is equivalent to the statement that $x > y \vee x = y \vee x < y$. The equivalence of the (classical) real numbers is, in its general form, impossible, provided one doesn't allow an infinite amount of evidence. From a constructivist point of view the equality of the (classical) real numbers is undecidable.†

This makes it clear that within the paradigm of discrete physics, one can allow computations involving the continuum, providing the Subject explicitly exhibits the crucial dependence on LPO. Recall that we are, primarily, dealing with the immediate *practice* of discrete physics (mathematics). This makes it possible to perform computations ("classical") within the paradigm of discrete physics (mathematics) without any loss of meaning and without any essential change in the method used. As Bishop puts it, "Classical mathematics would go on entirely as before except that every theorem would be written as an implication, either $LPO \rightarrow A$ or some extended version of an infinite computation implying A" [78].

Similarly, in discrete physics one can allow "classical" computations within the paradigm of discrete physics, provided one explicitly exhibits the dependence on LPO. Thus "classical" (continuum physics) can be regarded, when understood within the paradigm of discrete physics, as approximating discrete physics. However, the *canonical* formulation is to be performed by discrete physics adhering to constructive mathematics. This, among other things, shows the requirement of finite, and immediate, computability in discrete physics. As McGoveran puts it,

1) There is nothing in the knowable (or observable) Universe which cannot be described constructively.

2) There is nothing which can be described constructively which (that known as) the physical Universe can not produce (in a combinatorial sense).

3) There is nothing which can be observed or known which can not be described constructively.

This amounts, essentially, to the requirement of *finite reason* within the paradigm of discrete physics: the Subject is never in the immediate practice of physics to judge something which requires an infinite amount of evidence. Within the paradigm of discrete physics, consequently, there is no method requiring an infinite amount of evidence in order to solve of any problem A which option A or $\neg A$ we are justified in asserting prior to the actual verification of the task A. From this, one immediately notices the problem with the *Law of Excluded Middle*: its

† Note that this argument against LPO can not be given in a finite and discrete system such as the Ordering Operator Calculus since it requires postulating an infinite amount of potential evidence in order to satisfy the assumption stated above.

uninhibited use in immediate practice would lead to theories (programs) which one does not know how to execute (corroborate). Recall that a *law* is something that the Subject actually follows in immediate practice. There are no genuine tasks which require an infinite time (infinite amount of evidence) to be performed. Only when a computation (verification) is terminated can one claim a proposition to be factually true. There are no genuine laws prescribing an infinite amount of evidence. A law implicitly applied in immediate practice exhibits its finiteness in the very performance of the practice; there is nothing like an infinite practice. Every immediate practice (performative) is finite. Thus, a law only regulates a finite (immediate) routine. In order for this to be the case, the law must be meaningful, it must have a use within *both* the semantical as well as the syntactical part of the paradigm. As a law, "$A \vee \neg A$" clearly has no well-defined meaning within the paradigm of discrete physics in either case, when applied in an uninhibited way. It is never used (applied) in immediate (actual) practice. It is only used when *describing* a practice. The Law of Excluded Middle clearly has no use within the immediate *method* of discrete physics.‡

There cannot be, within the paradigm of discrete physics, any law which is impossible to apply in order to solve a certain task. The uninhibited use of the *Law of Excluded Middle* is not valid within the paradigm of discrete physics (and can be seen as a kind of "metaphor" according to Dummett), whereas the axiom of choice *is* valid [79].† The interesting consequence is that the axiom of choice, essentially, belongs to the paradigm of discrete physics, not only to mathematics. This is one of the basic reasons why it is preferable that theories of physics ought to be founded on the paradigm of discrete physics which has as its structural core The Intuitionistic Theory of Types (Sets). Whatever is computable and possible to corroborate within the paradigm of discrete physics, the *canonical* formulation exhibits a finite routine (method) for achieving this task.

It becomes, then, natural to assume that it ought to be feasible to extract from theories of physics, formulated within the paradigm of discrete physics, "expert programs" making "computer measurements" possible [80]. For this, one can use programming languages like *Pascal, C, LISP* and *PROLOG*. Recall, however, that in Martin–Löf's conception, mathematics *itself* is understood as exhibiting a high-level programming language. When extracting computer programs from constructive mathematics one can use Martin–Löf's Intuitionistic Theory of Types (Sets) as a *programming logic.* As Nordström points out, "Type theory can be seen as a programming logic, a logic for the process when programmers write a program for a certain task and give arguments why the program is correct. It is an important open problem in Computer Science whether it is feasible to use the computer not only for editing, storing and executing programs but also to check that the programs are correct .... Type theory suggests one way of doing this" [81]. This possibility has been explored by Constable et al. with the Nuprl Proof Development System [82], which is a concrete implementation of Martin–Löf's Type Theory as a programming logic. The system supports constructive type theory, whose primitive concepts can serve as building blocks for nearly any mathematical concept. This characteristic distinguishes Nuprl from most other proof-checking or theorem-generating systems. Nuprl runs in Zetalisp on Symbolics Lisp Machines and in Franz Lisp under Unix 4.2BSD.

‡ The Ordering Operator Calculus resolves this difficulty from the beginning since it is grounded upon finite tasks from the beginning. The Law of the Excluded Middle is therefore realized, and expressed, in an inhibited form only.

† Note, however, that the Ordering Operator Calculus need not and does not appeal to the axiom of choice, nor to prove it, since all finite orderings are well-orderings by definition. Recovering the continuum is not a goal of the Ordering Operator Calculus and, contrary to the position of Brower, it is a tenet of the theory that the continuum need not be recovered even in a constructive form.

Since discrete physics uses constructive mathematics (the ordering operator calculus), in the theories formulated within the paradigm, this means that the Type Theory also functions as a programming logic in discrete physics. One can say that it functions like a theory forming logic (programming logic) within the practice of discrete physics. This insight can now be extended to the practice of measuring a fact in physics. In discrete physics every theory (person program) exhibiting the canonical formulation, implicitly exhibits a logic for theory formation. In discrete physics (also in continuum physics) it is explicitly required that one can exhibit the theory (person program) to be tested. Thus, one can say that a theory of physics (person program) exhibiting the canonical form implicitly gives instructions for its own validity. Formulation of a theory in discrete physics always *intends* to achieve a reflective equilibrium between theory and the programming logic. It, furthermore, always attempts to achieve a reflective equilibrium between theory and fact measured within the paradigm—this, because the ultimate task of experimental physics is to exhibit (search for) the *factual structure* of the physical Universe. By factual structure is to be understood the judged, or corroborated (searched and found), facts of measurements.

In order to achieve this, it is not enough that the theory (person program) to be tested is formulated; one should also be able to exhibit a finite discrete method showing how to find the asserted fact. One way of formulating this is to say that every object of physics arrives equipped with its type-rule (Martin–Löf), or, alternatively, to say that every proposition arrives equipped with its verification procedure (proof, computation). Verifications (computations) and corroborations (measurements) are built into the practice. This is an essential characteristic of McGoveran's Ordering Operator Calculus.

This leads to the insight that practice of physics can be understood according to the idea of *Phenomenon–As–Games* .

In order to be able to grasp this point more easily, it is useful to compare this to the way Martin–Löf explains the analogous case of mathematical practice: the "propositions-as-types" idea. First, the method of Martin–Löf exhibits a number of *categories*. So also in prephysics. Recall that due to **TAP** one cannot analyze language (as the universal medium of communication) with the help of any category, since all categories only appear in language. The word "category" is used here in its older, philosophical sense, not in the modern sense of "category theory." To exhibit a category the Subject just has to tell "what a thing is." That is, as we said above, every object (of knowledge) arrives equipped with its type (set, category).

There are objects, since a judgment (operation, assertion) can be understood either as an *act* of judging, or, as an *object* of knowledge. However, due to philosophical nonseparability, these are not two separate entities: the act of judging and the object of judgment *fuse*. To judge an expression A to be a proposition, one must carry out the judging of A. Thus, if A is a proposition, we know (implicitly) what to do in order to tell what its canonical proof is; we know *how* to exhibit A. For example, in Martin–Löf's method, the set of integers $\mathbf{N}$ can be thought of as the proposition "there is a natural number." Now, any exhibited integer constitutes a direct (immediate, canonical) proof that there is a natural number. If the notion of "proposition" is understood in this general way, we can, for example, render the category "$a \in A$" as (1) "a is a member of the type (set) A," or (2) "a is a proof of the proposition A," as Beeson formulates this point [83]. Discrete physics is to be understood in a similar way, when read along the insight provided by the "phenomenon-as-games" idea.

If we understand the word "proposition" in this way, one ought not to have any difficulty with the equivalence of a phenomenon and a game (measurement). Following Bohr's terminology, one can say that a phenomenon and a game (measurement) exhibit *complementary* readings of a judgment. That is, this point exhibits the nonseparability (i.e., the unity of method and fact obtained) as it occurs in Bohr's characterization of the impossibility of any sharp separation between the behavior of atomic objects and the interacting with the measuring instruments which serve to define the conditions under which the phenomena appear. One of the insight's to be gained is that one can now realize that Bohr understood physics along the lines provided by the view of "language as the universal medium" and that a "complementary reading" belongs to the conditions that the transcendental Subject lays down for the empirical Subject to fulfil.

Theories (person programs) formulated within the paradigm of discrete physics can be given a number of complementary readings. Assume a to stand for a theory (person program) of physics and A to stand for a proposition. Then one can provide, at least, the following readings. The names occurring within parentheses express the corresponding readings in the practice of mathematics (presupposed known), within the paradigm, except for the last one. As examples one can provide the following:

1) a is a solution to the problem A (Kolmogorov).

2) a is a program for the specification of A (Martin–Löf).

3) a is a method of fulfilling (realizing) the intention (expectation, task) A (Heyting, Husserl).

4) a is a (winning) strategy for the game A (Hintikka, Ranta).

5) a is an instantiation (realization, instance) of the constructive (recursive) model A (Kleene, Rogers, McGoveran).

6) a is a measurement of the phenomenon A (Bohr, Wheeler, Gefwert).

The system is an inherently open system, in the sense that it is possible to extend it with new program forming (complementary) expressions and new type (set) forming operations.

We shall now investigate the last reading in somewhat more detail. As far as the practice of physics is concerned, one can give the formalism of theoretical physics, a genuinely novel and discrete reading, in accordance with the idea of a game (measurement) as exhibiting a phenomenon. Here one could use Ryle's terminology and say that one reads the practice of physics as exhibiting a genuine "game of exploring the world." The world or, more appropriate, nature is then understood as exhibiting the transcendental rules of the game (The Transcendental Anthropic Principle). The meaning of any practice of measuring in physics (implicitly incorporating the theory (person program) to be falsified or corroborated by virtue of the *result* of the measurement) can only be grasped provided the practice takes place in the life-world (*Lebenswelt*). Recall that the Subject is a participator in the factual investigation of nature performed in the demonstrative practice of physics. The notion of a "measurement" is to be understood as promoting the observer to participator status; i.e., it is "built into" the paradigm of (discrete) physics.

A phenomenon is only established as the result of engaging in a measurement verifying a proposition in accordance with certain rules. Thus, the practice of measuring a proposition to be true (truth-as-fact) can be read as a game corroborating (verifying) a proposition to be a phenomenon in Bohr's and Wheeler's sense. What is important is that a certain result is achieved (by virtue of some solution, program, method, winning strategy or measurement) and that the Subject understands it to be achieved. When the Subject has formulated (within the paradigm) a theory (person program) of physics and it is corroborated by virtue of the *result* of a measurement, it has formulated:

1) a solution to a certain problem A;

2) a method fulfilling an intention of corroborating A;

3) a (winning) strategy of searching and finding A;

4) a measurement establishing the factual structure of the phenomenon A.

As far as meaning is concerned, these linguistic forms exhibit redundant ways of expressing the same point: an effective strategy (method) implicitly applied in order to corroborate (verify) the existence of a proposition by *"immer ausfurbare Operationen,"* as Gödel formulates the point in his *Dialectica* paper [84]. This has not been generally understood.

In setting up the code (the R–frame and the P–frame) one is implicitly providing a translation (or modelling) between those frames and informal linguistic forms occurring in the E–frame. A certain judgment (category) may translate into several different forms of informal language (the E–frame). The translation manual (the P–frame) enables one to determine how far the ordinary forms of expressions (occurring within the continuum paradigm) can be formulated within the discrete paradigm setup intended as a code of the practice of physics without applying *LPO*. The translation manual (or modelling) enables one to grasp how far one's ordinary forms of expression may be given a meaningful explanation within the paradigm set up as a code of the practice of physics; that is, there are often many different ways in which a single judgment is expressed in physics. This may constitute a redundancy which the code eliminates.

It is important to realize that the translation manual is meant as a semantical explanation of how the practice is to be understood. This is precisely what one performs when engaging in semantics. To assume that the translation manual would provide the explanation of meaning, *presupposes* that the expressions occurring within the continuum paradigm are the ones determining how the expressions are to be understood. Not so. The aim with semantics is to convey how the expressions occurring in the E–frame *are to be* understood. Note that the practice of physics can only evade a verification transcendent semantics when understood within the semantical -paradigm of "language as the universal medium." This possibility is excluded within the semantical paradigm of "language as calculus" (model theory).

Discrete physics exhibits the basic constructivist tenet: when one asserts that an object of knowledge exists (having certain desired properties), one should be able to show how to find it by using a finite routine (theory of physics, person program) in such a way that a computer suitable programmed can verify (in normal science) the result. It may be, as emphasized by Truesdell in a lecture in Milan, that the computer may have an impact on mathematics comparable to that which the microscope had on biology and the telescope on astronomy. The understanding arising from the paradigm of discrete physics is that the computer, eventually, will be as influential in theoretical physics as it will be in mathematics. However, this requires that the semantical part as well as the syntactical part of the paradigm of physics is to change, so that the immediate practice of physics *can be made more meaningful* than it is when performed in virtue of the current paradigm accepting the continuum. This requires revolutionary changes, conceptually (unification of quantum theory and general relativity on a discrete basis) as well as in the mathematics used in theoretical physics. It is against this background, and **only** when it is achieved, that one is to understand Hawking's conjecture that "At present computers are a useful aid in research but they have to be directed by human minds. However, if one extrapolates their recent rapid rate of development, it would seem quite possible that they will take over altogether in theoretical physics." However, even within the paradigm of discrete physics, the computer will never take over from the judging Subject: only physicists judge whereas computers do not. Verification and corroboration exhibit judgments.

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REFERENCES

1. See, for comparison, Torraldo diFrancia, G., *The Investigation of the Physical World,* Cambridge (1981), p. 10.
2. Newton, I., *Philosophiae Naturalis Principia Mathematica,* 3rd ed. (1726), Vols. I–II, assembled and edited by Koyré, A and Cohen, I. B., Cambridge (1972).
3. Kuhn, T. S., *The Structure of Scientific Revolutions,* 2nd ed., Chicago (1970), p. 11.
4. Ibid., p. 11.
5. Bastin, T., *Quantum Theory and Beyond,* Cambridge (1971).
6. McGoveran, D., *"Foundations for a Discrete Physics,"* in *Proc. ANPA9, 1988* Noyes, H. P., ed.
7. Poincaré, H., *The Foundations of Science,* Lancaster (1946), p. 281.
8. Ibid., p. 284.
9. Dijkstra, E. W., *A Discipline of Programming,* New Jersey (1976), p. 201.
10. Polanyi, M., *Science 113* (1968), p. 1308.
11. Bishop, E., *Foundations of Constructive Analysis,* New York (1967).
12. Martin–Löf, P., *Intuitionistic Type Theory,* Notes by Giovanni Sambin of a series of lectures given in Padua, June 1980, Bibliopolis, Naples (1984).
13. Beeson, M. J., *Foundations of Constructive Mathematics,* Berlin, Heidelberg, New York, Tokyo (1985), p. 282.
14. Martin–Löf, P., *"Constructive Mathematics and Computer Programming,"* in *Logic Methodology and Philosophy of Science VI,* Amsterdam (1982), p. 156.
15. Ibid., pp. 154–6.
16. See Ref. 6.
17. Popper, K. R., *The Logic of Scientific Discovery,* London (1972), Sixth Impression, p. 59.
18. Kuhn, T. S., *The Structure of Scientific Revolutions,* 2nd ed., Chicago (1970), p. 176.
19. Polanyi, M., *Personal Knowledge,* Chicago (1958).
20. Noyes, H. P., *"DISCRETE PHYSICS: Practice, Representation and Rules of Correspondence,"* in Proc. ANPA 9, Noyes, H. P., ed.
21. Ibid.
22. Wheeler, J. A., *"Bits, Quanta, Meaning,"* in *Problems in Theoretical Physics,* Giovanni, A., Mancini, F., and Marinaro, M., eds., Univ. of Salerno Press, Salerno (1984), pp. 121–141.
23. Martin–Löf, P., *Intuitionistic Type Theory,* Bibliopolis (1984), p. 71.
24. Poincaré, H., *The Foundations of Science,* Lancaster (1946), p. 40.
25. Weyl, H., *"Comments on Hilbert's Second Lecture on the Foundations of Mathematics," 1927, pp. 482–3,* reprinted in *From Frege to Gödel. A Source Book in Mathematical Logic, 1879-1931,* van Heijenoort, J., ed., Cambridge, Massachusetts (1981), pp. 480–4.
26. Kuhn, T. S., *The Structure of Scientific Revolutions,* 2nd ed. (1970), p. 77.

27. Martin–Löf, P., *"Constructive Mathematics and Computer Programming,* in *Logic, Methodology and Philosophy of Science VI,* Amsterdam (1982), p. 166.

28. Rorty, R., *Philosophy and the Mirror of Nature,* Princeton, New Jersey (1979), p. 17.

29. Ryle, G., *The Concept of Mind,* Hammondsworth, Middlesex (1973), p. 17.

30. Ibid., p. 23.

31. van Heijenoort, J., *Logic as Language and Logic as Calculus,* Synthese, Vol. 17 (1967), pp. 324–30.

32. Hintikka, M. B. and Hintikka, J., *Investigating Wittgenstein,* Oxford (1986), p. 2.

33. Barrow, J., D. and Tipler, F. J., *The Anthropic Cosmological Principle,* Oxford (1986), p. 472.

34. Patton, C. M. and Wheeler, J. A., *"Is Physics Legislated by Cosmogony?,"* in *Quantum Grairty, An Oxford Symposium,* Isham, C. J., Penrose, R. and Sciama, D. W., eds., Oxford (1975), p. 560.

35. Ibid., p. 562.

36. Herbert, N., *Quantum Reality, Beyond the New Physics,* New York (1985), p. 158.

37. Wheeler, J. A., *How Come the Quantum?,* p. 313, in *New Techniques and Ideas in Quantum Measurement Theory,* Greenberger, D. M., ed., N. Y. Acad. Sci., Vol. 480., New York (1986).

38. Hintikka, M. B. and Hintikka, J., *Investigating Wittgenstein,* Oxford (1986), p. 3.

39. Husserl, E., *Cartesion Meditations,* Cairns, D., tr., The Hague (1973), p. 5.

40. Bohr, N., *Atomic Physics and Human Knowledge,* New York (1958), p. 89.

41. Petersen, A., *Quantum Physics and the Philosophical Tradition,* Cambridge, Mass. (1968), p. 172.

42. Dicke, R. H., *Rev. Mod. Physics 29,* pp. 355, 363; Nature *192* (1961), p. 440.

43. Carter, B., in *Confrontation of Cosmological Theories with Observation,* Longhair, M. S., ed., Dordrecht (1974), p. 291.

44. Wheeler, J. A., *Genesis and Observership,* in *Foundational Problems in The Special Sciences,* Butts, R. E. and Hintikka, J., eds., Dordrecht, Boston (1975), p. 3.

45. Whitrow, G., Br. J. Phi. Sci. *6* (1955), p. 13.

46. McGoveran, D., *"Foundations for a Discrete Physics,"* in *Proc. ANPA9, 1988* Noyes, H. P. ed.

47. Hintikka, M. J. and Hintikka, J., *Investigating Wittgenstein,* Oxford (1986), p. 1.

48. Ibid., pp. 1–2.

49. Petersen, A., *Quantum Physics and the Philosophical Tradition,* Cambridge, Mass. (1968), p. 172.

50. Ibid., p. 188.

51. Wittgenstein, L., *Philosophical Grammar,* Rhees, R. R., ed., Kenny, A., tr., Oxford (1974), p. 462.

52. Husserl, E., *Cartesian Meditations,* Cairns, D., tr., The Hague (1973), p. 26.

53. Ibid., p. 84.

54. Sundholm, G., *Constructions, Proofs, and the Meaning of the Logical Constants,* J. Phil. Logic. *12* (1983), pp. 151–172.

55. Husserl, E., *Formal and Transcendental Logic,* Cairns, D., tr., The Hague (1969), pp. 313–4.

56. See Ref. 44, p. 29.

57. Wittgenstein, L., *Tractatus Logico–Philosophiens,* London (1961), § 5.632.

58. Ibid., § 5.621.

59. Ibid., § 5.64.

60. Wittgenstein, L., *Philosophical Remarks,* Rhees, R., ed., Hargreaves R. and White, R., trs., Oxford (1975), p. 81.

61. Patton, C. M. and Wheeler, J. A., *Is Physics Legislated by Cosmogony?,* p. 560. cf., n. 34.

62. Ryle, G., *"Sensations,"* in *Contemporary British Philosophy,* Third Series, Lewis, H. D., ed., London (1956), p. 442.

63. Prawitz, D., *Dummett on a Theory of Meaning and its Impact on Logic,* University of Stockholm Preprint, Department of Philosophy (1987), p. 39.

64. Einstein, A., *"On the Method of Theoretical Physics,"* The Herbert Spencer lecture delivered at Oxford, June 10, 1933, Oxford, 15 pp.

65. Wittgenstein, L., *Culture and Value,* von Wright, G. H. and Nyman, H., eds., Winch, P., tr., Oxford (1980), p. 27.

66. Wittgenstein, L., *On Certainty,* Anscombe, G. E. M. and von Wright, G. H., eds., with an Introduction by von Wright, G. H., Oxford (1969), § 105.

67. Einstein, A., *Theory of Relativity: Its Formal Content and Its Present Problems,* Rhodes lectures at Oxford University, May 1931, unpublished (briefly described in *Nature,* Vol. 127, pp. 765, 790, 826–27).

68. Wittgenstein, L., *"Remarks on the Foundations of Mathematics,"* von Wright, G. H., Rhees, R. and Anscombe G. E. M., eds., Anscombe, G. E. M., tr., Oxford (1978), 3rd ed., § 118.

69. Diamond, C., ed., *Wittgenstein's Lectures on the Foundations of Mathematics 1939,* from the notes of Bosanquet, R. G., Malcolm, N., Rhees, R. and Smythies, Y., Cornell, Ithaca, New York (1976), p. 230.

70. For comparison, see Hintikka, M. J. and Hintikka, J., *Investigating Wittgenstein,* Oxford (1986), pp. 21–2, for a commentary on this quotation from an unpublished MS by Wittgenstein (MS 109, p. 58).

71. Wittgenstein, L., *Philosophical Grammar,* Rhees, R., ed., Kenny, A., tr., Oxford (1974), p. 111.

72. See, for comparison, Noyes, H. P., McGoveran, D., Etter, T., Manthey, M. J. and Gefwert, C., *A Paradigm for Discrete Physics?* Section 8: Foundations of Physical Sciences of the 8th International Congress of Logic, Methodology and Philosophy of Science, Moscow, USSR, August 17–22 (1987); also see Noyes, H. P., Gefwert, C., and Manthey, M. J.,

"A Research Program with No 'Measurement Problem,'" in Greenberger, D. M., ed., *New Techniques and Ideas in Quantum Measurement Theory,* Annals of the New York Academy of Sciences, Vol. 480 (1986), pp. 553–563.

73. Dummett, M., *"The Philosophical Basis of Intuitionistic Logic,"* in *Truth and Other Enigmas,* London (1978), p. 217.
74. Prawitz, D., *Dummett on a Theory of Meaning and Its Impact on Logic.* cf., n. 63, p. 7.
75. Petersen, A., *Quantum Physics and the Philosophical Tradition,* Cambridge, Mass. (1968).
76. Hintikka, J. and Kulas, J., *The Game of Language,* Dordrecht (1983).
77. Nordström, B., *"Martin–Löf's Type Theory as a Programming Logic,"* Notes for a short course consisting of three lectures given at the X Incontro di Logica Matematica in Siena, Italy April 2–5, 1986, p. 1, Programming Methodology Group, Chalmers University of Technology and University of Göteborg (1986).
78. Bishop, E., *Foundations of Constructive Analysis,* New York (1967).
79. Martin–Löf, P., *"Constructive Mathematics and Computer Programming,"* in *Sixth International Congress for Logic, Methodology and Philosophy of Science,* Amsterdam (1982), p. 156.
80. Hut, P. and Sussman, G. J., *Advanced Computing for Science,* Scientific American, October 1987, pp. 137–144.
81. Nordström, B., *Martin–Löf's Type Theory as a Programming Logic,* cf., n. 77, p. 2.
82. Constable, R. L. et al., *Implementing Mathematics with the Nuprl Proof Development System,* New Jersey (1986).
83. Beeson, M. J., *Foundations of Constructive Mathematics,* Berlin, Heidelberg, New York, Tokyo (1985), p. 247.
84. Gödel, K., *Über eine bisher noch nicht benützte Erweiterung des finites Standpunktes,* Dialectica *12* (1958), pp. 280–7.